BASIC MATHEMATICS

→ Basic mathematical knowledge gives us the easy way of understand the science (nature) and technology (core).

→ It is the key subject for all science & tech subjects.

✓ it has "Numbers & decimal points, Algebra (+, −, ÷, ×) & Homogeneity, Variable & constants, equalities (=), BODMAS rule, Ratio, Average, percentage, increase & decreases, Trigonometry, Geometry (different shapes), power of 10, function & equation, proportionality & graphs."

Numbers & Decimal points:

We live on our mother earth, and we are using all the resources on earth which is available in the form of material or Matter those are solids, liquids, Gases. clearly there is no numbers on earth there is Matter and energy only.

For counting the similar types of Matter (bodies), we have to used the easily understandable key that is number → Mathematical Language.

Numbers (we create it)

- Integers (Full Material) → 1, 2, 3, 4, ------ → 1st Apple, 2nd Apple, 3rd Apple ---
- Decimal (part of the Material) → $\frac{1}{2}, \frac{1}{4}, \frac{1}{10}$, ------ → half Apple (0.5), Quarter Apple (0.25) ------

Q). How many values in between 1 and 2 ?

Answer → ∞ (infinity) → it depends on number of parts.

Ex: if it is "1cm" (centimeter) → it has 10 parts → 10 values
if it is "m" (meter) → it has 100 parts → 100 values
if it is "km" (Kilometer) → it has 1000 parts → 1000 values.
if it is "minute" → it has 60 parts → 60 values.

Ex-1 mass of the body $1\,Kg = 1000\,gm \longrightarrow 1\,gm = \frac{1}{1000}\,Kg = 0.001\,Kg$.

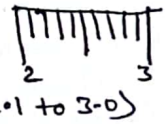
0.001 Kg (1 gm), 0.002 Kg (2 gm), ---, 0.100 Kg (100 gm), --- 0.250 Kg (250 gm ($\frac{1}{4}$ Kg)), --- 0.500 Kg (500 gm ($\frac{1}{2}$ Kg)), --- 0.750 Kg (750 gm ($\frac{3}{4}$ Kg)), --- 0.999 Kg, 1.000 Kg (1 Kg)

That is 1 Kg having 1000 parts.

Ex-2

Length → $1\,cm = 10\,mm \longrightarrow 1\,mm = \frac{1}{10}\,cm = 0.1\,cm$.

0.1 cm (1mm), 0.2 cm (2mm), ---- 0.9 cm (9mm), 1.0 cm (10mm) → 10 parts.



(2.1 to 3.0)
(2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0)

Length → 1 feet = 12 inch → 12 parts.

A Man of height → 5.1 → 5ft & 1 inch

→ 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 5.10, 5.11, 5.12 (OR) 6 ft.

Ex:3 Money → 1 Rupee = 100 paise
→ 0.1 paise, 0.2, 0.3 - - - 0.25 - - - 0.50 - - - 0.99, 0.100 paise (or) 1 Rupee.

Ex:4 Cricket → 1st over = 6 balls.
→ 1st ball = 0.1 over
2nd ball = 0.2 over
6th ball = 0.6 over → 1 over completed.
2nd over, 1st ball = 1.1 over
2nd over, 2nd ball = 1.2 over - - - - .

Ex:5 Age → born baby age → 1 Month → 0.1 year
2 Month → 0.2 year
12 Month → 0.12 year (or) 1 year completed.

2. Algebra (+, −, ×, ÷, x^2, x^3, $\sqrt{x}$, $\frac{1}{x}$ - - - - -)

Addition & Subtraction :- (+, −)

in mathematics → $5+3=8$, $5-3=2$, $a+b=b+a$, $a-b \neq b-a$

but in physics → It is Homogeneity principle → 1st Revolutionary point in Science
→ Add, sub can possible between two similar quantities.
→ 5 kg + 3 sec → No Answer
→ 5 kg + 3 kg = 8 kg ✓ weight$_1$ + weight$_2$ = Total weight
5 kg − 3 kg = 2 kg ✓
4 sec + 10 sec = 14 sec ✓
10 Newton + 20 Newton = 30 Newton (Total force) ✓

$x + 8\,\text{sec} = 12\,\text{sec}$
$x = 12 - 8 = 4\,\text{sec}$

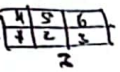
$2 - x = 10$
$x = 2 - 10$
$x = -8$

$2x + 10 = 14$
$2x = 4$
$x = \frac{4}{2} = 2$

Multiplication (×) :- (it gives 2D, 3D answers)

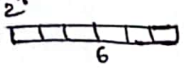
No rule for this, we can multiply any two quantities to get 3rd one.

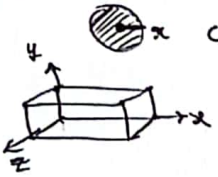
$6 \times 4 \rightarrow 24 \rightarrow 4+4+4+4+4+4$ (or) $6+6+6+6$

2cm × 3cm → 6 cm^2 → Area given →  → $6 \times 1\,cm^2$

$a \times b = b \times a$

3cm × 2cm → 6 cm^2 → 

6cm × 1cm → 6 cm^2 → 



circle Area = $\pi r^2 \rightarrow cm^2$
$\pi = 3.14 = 22/7$

Rectangular cube
Volume = Area × height
$= l\,b\,h$
$= x\,y\,z \rightarrow cm^3$

The two quantities may (or) may not be the same quantities.

Ex:1 Force (F) = mass (m) × acceleration (a)
$F = 2\,Kg \times 10\,m/sec^2 \rightarrow 20\,Kg \cdot \frac{m}{sec^2}$
$F = 20$ Newton.

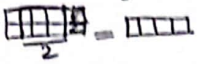
Ex:2 distance travelled = speed × time of travel
$d = v \times t$
$= 40\,\frac{km}{hour} \times 2\,hr$
$d = 80$ km travelled.

Ex:3 $x \times 2\,sec = 40$ metre
$x = \frac{40}{2} = 20\,m/sec \rightarrow$ velocity.

$x + 0 = x$	$x \times \infty = \infty$
$x - 0 = x$	$\frac{x}{\infty} = 0$
$0 - x = -x$	$\frac{\infty}{x} = \infty$
$x + \infty = \infty$	$\frac{x}{0} = \infty$
$x - \infty = -\infty$	$\frac{0}{x} = 0$
$\infty - x = \infty$	

division (a/b):

→ Mathematically → $\frac{8}{2} = 4$, Physics → $\frac{8\text{ Km}}{2\text{ hr}} \rightarrow 4\text{ Km/hr}$



General knowledge → $\frac{8 \text{ bananas}}{2 \text{ rupees}} \rightarrow \frac{4 \text{ bananas}}{1 \text{ rupee}}$.

⇒ $\frac{1.562}{2} = 0.781$, $\frac{3.562}{2} = 1.781$, $\frac{5.562}{2} = 2.781$

→ $\frac{1}{0.1} = \frac{1}{1/10} = 10$, $\frac{1}{0.01} = 100$

 1 Apple → 100% ,  $\frac{1}{2}$ Apple → 50% ,  $\frac{1}{3}$ apple, 25%

$\frac{3}{4}$ apple, 75%.

→ $(0.2)^2 = \left(\frac{2}{10}\right)^2 = \frac{4}{100} = 0.04$

→ $(0.4)^2 = \left(\frac{4}{10}\right)^2 = \frac{16}{100} = 0.16$

→ $x^m \times x^n = x^{m+n}$ → $x^3 \times x^6 = x^{3+6} = x^9$

→ $\frac{x^m}{x^n} = x^{m-n}$ → $x^8/x^2 = x^{8-2} = x^6$

→ $(x^m)^n = x^{mn}$ → $(x^2)^3 = x^{2\times3} = x^6$, $\left(x^8\right)^{\frac{1}{2}} = x^{8/2} = x^4$

$\frac{a/b}{c} = \frac{a}{b \times c}$

$\frac{a}{b/c} = \frac{a \times c}{b}$

$\frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$

→ equality ⇒ $x^a = x^b$ then $a = b$ → if bases equal then powers may equal

⇒ $a^x = b^x$ then $a = b$ → if powers equal then bases may equal.

→ $\frac{1}{x^n} = x^{-n}$, $\frac{1}{x^m} = x^m$, $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

→ $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{3} = 1.732$, $\sqrt{2} = 1.414$, $\frac{1}{\sqrt{2}} = 0.707$, $\frac{\sqrt{3}}{2} = 0.866$

→ $4 \times 0.01 = 0.04$
$74 \times 0.01 = 0.74$
$374 \times 0.01 = 3.74$

$30 \times 0.1 = 3$
$30 \times 0.01 = 0.30 = 0.3$
$30 \times 0.001 = 0.03$

$0.4 \times 10 = 4$
$0.4 \times 100 = 40$
$0.4 \times 1000 = 400$
$0.43 \times 100 = 43$.

→ $\frac{2}{10} = 0.2$, $\frac{2}{100} = 0.02$, $\frac{2}{1000} = 0.002$, $\frac{23}{10} = 2.3$, $\frac{23}{100} = 0.23$

3. variable & constant.

Variable → The value of the quantity may change (increase, decrease).

→ denoted by alphabets → x, y, z, p, q, r, s, d, t, F - - - - - - .

→ Pen is constant, ink is variable.

→ battery voltage is constant, current comes out is variable.

→ bottle volume is constant, liquid in it is variable.

Ex: $3x + 4y = 15$
3, 4, 15 → constants
x, y → variable

, $y = 6x + 10$ like $y = mx + c$ → Straight line equation.
input $x = 0, 1, 2, 3 - - -$
output $y = 10, 16, 22 - - - - -$.

Constant → The quantity which does not change.

→ denoted by numbers & proportionality constants.

→ density of a material, ~~Resistance~~ Resistivity of the metal, Volume, Gravitational constant, gas constant, permeability of free space, permittivity of Air,

↳ see page NO:

Ex: $F = m \cdot a$
F → Force → hitting bat → variable
m → mass → ball/bat → constant
a → acceleration → change in speed of a ball → variable

4. BODMAS Rule

→ All the mathematical calculations in any science & Tech subjects can be done by this rule.

→ calculators, computers, → works on this rule in software programming.

B O D M A S → Subtraction

↓ Bracket ( ) ↓ of ↓ Division ÷ ↓ Multiplication × ↓ Addition +

Ex: 1) two plus two by two → $2+\frac{2}{2} \rightarrow 2+1 \rightarrow 3$ ✓ (correct)

→ $\frac{2+2}{2} \rightarrow \frac{4}{2} \rightarrow 2$ ✗ (wrong)

In the calculator, we can type it as → $2+(2\div 2)= \rightarrow 3$

→ $(2+2)\div 2= \rightarrow 2$

2) $4+(16\div 2-1)3 = 4+(\frac{16}{2}-1)\times 3$

$= 4+(8-1)\times 3 = 4+(7\times 3) = 4+21 = 25$ ✓

5. Ratio

→ comparision between two variables.

$a:b = 3:2 \rightarrow a=3x,\ b=2x$

$\rightarrow \frac{a}{b}=\frac{3}{2} \rightarrow a=1.5\times b$

→ a > b, 1.5 times.

Ex: 1) Boys : Girls = 60 : 30

B : Gr = 2 : 1 → $\frac{B}{Gr}=\frac{2}{1} \rightarrow B=2Gr.$

2) salt : chilli = 1 : 3 → $\frac{S}{C}=\frac{1}{3} \rightarrow S=\frac{C}{3}$ (or) $C=3S$

3) Sand : cement = 4 : 1 → $\frac{S}{C}=\frac{4}{1} \rightarrow S=4C$

6. Average value

→ It is the moderate (medium) value between high (Max) & Low (Min).

→ Ave = $\frac{\text{sum of elements}}{\text{no. of elements}} = \frac{b}{a}$ → it gives no. of 'b' values per one 'a' value.

→ it tells the equalization.

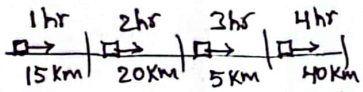
→ Movie review → worst (Min), it's OK (Ave), super (Max)

→ Taste of food → waste (Min), OK (Ave), WOW (Max)

Ex: 1) 3 childrens having biscuits 6, 2, 10 → Mother sharing equally as

→ Ave = $\frac{6+2+10}{3} = \frac{18}{3} = 6$ biscuits / 1 children

2) A car moving as



The Average speed $V = \frac{\text{Total distance}}{\text{Total time travel}}$

$= \frac{15+20+5+40}{4} = \frac{80}{4} = 20$ km/hr

3)

S.No	diameter of sphere
1	2.52 cm
2	2.56 cm
3	2.57 cm → Max
4	2.51 cm → Min
(5)	2.54 cm
	12.70 cm

Ave diameter $d = \frac{12.70}{5} = 2.54$ cm.

7: Percentage (%) value

→ It is the value per every 100.

$1 \to 100\%$, $\frac{1}{2} \to 50\%$, $\frac{1}{3} \to 33.3\%$, $\frac{1}{4} = 0.25 \to 25\%$

$\frac{3}{4} = 0.75 \to 75\%$, $2 \to 200\%$, $3 \to 300\%$, $\frac{1}{10} = 0.1 \to 10\%$

Doubled $\to 2x \to 200\%$, tripled $\to 3x \to 300\%$

halfed $\to \frac{x}{2} \to 50\%$ Quartered $\to \frac{x}{4} \to 25\%$

Ex:1 The cost of a dress is 400/- with 10% discount.

∴ The Amount to be paid $= 400 - (400 \times 10\%)$

$= 400 - 40 = 360$ rupees.

Ex:2 In a classroom 48 students passed out of 60,

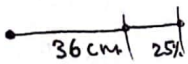
Then pass% is $= \frac{48}{60} \times 100 = 8 \times 10 = 80\%$.

Ex:3 Tooth paste original weight is 150 gm, In an offer → 20% extra.

then the final weight $= 150 + (20\% \text{ of } 150)$

$= 150 + (150 \times \frac{20}{100}) = 150 + 30 = 180$ gms.

Ex:4 A thread having length 36 cm. if another thread of 25% is added then final length is —.



Total length $= 36 + (36 \times \frac{25}{100})$

$= 36 + (\frac{36}{4}) = 36 + 9 = 45$ cm.

formula → $l + (l \times \frac{25}{100})$

$\to l + \frac{l}{4} \to \frac{5l}{4}$.

Ex: Net salary = Basic + 15% DA + 10% HRA.

8: increases & Decreases

Some times the value of variable may increase as $x, 2x, 3x, 4x$ ----- (100%, 200%, 300%, ------)

it may decrease as $\frac{x}{2}, \frac{x}{3}, \frac{x}{4}, \frac{x}{5}$ ------- (50%, 33%, 25%, 20%)

$\frac{x}{25} \to 4\%$, $\frac{x}{10} \to 10\%$, $\frac{3x}{4} \to 75\%$, $\frac{3x}{2} \to 150\%$, $\frac{2x}{3} \to 66\%$, $\frac{5x}{2} \to 250\%$

9: Different shapes

Two dimensional shapes → (2D) → Rectangle → paper, Black Board, plank ...

→ square → TV screen, paper, Napkin.

Three dimensional shapes → 3D → Triangle → samosa, Ladder on wall, Chips, steps

(3D) → cubes → Book, Match Box, Lunch Box, Soap, TV, Cake, Bed, Bus, Eraser.

(2D) → circle → FAN, Motor, Wheels, Orbit, coin, Dosa,

(3D) → sphere → Ball, Fruits, planets, orbital, Eyeball,

(3D) → cylinder → pen, pencil, Bottle, Snake, Pipes, Bullet

(3D) → cone → cyclon, Icecream Cone,

10: power of 10 (Multiplier)

Some times the value of a quantity may increase (or) decrease with respect to 10 as,

10 times increase → $3\times10 \rightarrow 30 \rightarrow 3\times10^{1} \rightarrow 3+3+3+3+3+3+3+3+3+3$

100 times increase → $3\times100 \rightarrow 300 \rightarrow 3\times10^{2}$

1000 times increase → $3\times1000 \rightarrow 3000 \rightarrow 3\times10^{3}$

10 times decrease → $\frac{3}{10} \rightarrow 0.3 \rightarrow 3\times10^{-1}$

100 times decrease → $3/100 \rightarrow 0.03 \rightarrow 3\times10^{-2}$

1000 times decrease → $3/1000 \rightarrow 0.003 \rightarrow 3\times10^{-3}$, In general it has some ranges name

it will be used in all over the world in science & Technology.

da → deca → 10^{1}

h → hecta → 10^{2}

K → Kilo → 10^{3} → 1000
10^{4} → 10000
10^{5} → 1,00,000 → 1 lakh (India)

M → Mega → 10^{6} → 10 lakh → 1 Million (world) ✓
10^{7} → 100 lakh → 1 crore (India)
10^{8} → 10 crore

G → Giga → 10^{9} → 100 crore → 1000 Million
→ 10000 lakh → 1 Billion (world) ✓

T → Tera → 10^{12} → 1 lakh crores → 1000 Billion
→ 1 Trillion (world) ✓

P → Peta → 10^{15} → 1000 Trillion

E → Exa → 10^{18}

Z → Zetta → 10^{21}

Y → Yotta → 10^{24}

increase ↑

d → deci → $10^{-1} \rightarrow \frac{1}{10} \rightarrow 0.1$

centi → c → $10^{-2} \rightarrow \frac{1}{10^{2}} \rightarrow \frac{1}{100} \rightarrow 0.01$

m → milli → $10^{-3} \rightarrow \frac{1}{1000} \rightarrow 0.001$
→ 10^{-4}
→ 10^{-5}

μ → micro → 10^{-6}

n → nano → 10^{-9}

p → pico → 10^{-12}

f → femto → 10^{-15}

a → atto → 10^{-18}

z → zepto → 10^{-21}

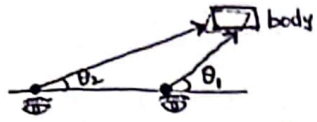
y → yocto → 10^{-24}

↓ decrease

examples

1) Mobile camera resolution is denoted in Megapixels → 16MP, 8MP ----
2) Photo Memory allocation → in KB (Kilobytes), MB (Megabytes).
3) Movie Memory space in → 700 MB (DVD quality), 1.6 GB (HD quality).
4) Computer RAM, ROM (Harddisk) Memory → 4 GB RAM, 500 GB ROM.
5) electron charge $e = 1.6\times10^{-19}$ coulomb, Mass $m = 9.1\times10^{-31}$ kg.
6) Avagadro number $n = 6.023\times10^{23}$ molecules/mole.
7) planck's constant $h = 6.624\times10^{-34}$ Joule × sec.
8) Resistance of the Resistor $R = 50$ KΩ (Kiloohms), $= 50\times10^{3}$ Ω
100 MΩ (Megaohms). 100×10^{6} Ω.
9) Electric current denoted by I → A, mA, μA, 5mA → 5×10^{-3} → 0.005 Amp
20 μA → 20×10^{-6} Amp.
10) Capacitance of a capacitor C = 0.1 μF (microFarad)
C = 10 pF (picoFarad).
11) Vehicles travelled distance d = 40 Km (Kilometer) = 40×10^{3} meter = 40,000 metre
12) Light velocity $c = 3\times10^{8}$ m/sec = $3\times10^{5}\times10^{3}$ m/sec = 3×10^{5} Km/sec = 3,00,000 Km/sec
c = 3 lakh Km/sec.
13) Light year (d) = distance travelled by light in 1 year in the universe.
$d = c\times t = 3\times10^{8}\ \frac{m}{sec} \times 1$ year $= 3\times10^{8}\ \frac{m}{sec} \times 365\times24\times3600$ sec
$d = 94608\times10^{11}$ meter $= 94608\times10^{8}$ Km

→ No two persons can see an object equally → 2nd Revolutionary point in science.



→ It gives us the idea of Angle (θ) of site.
→ Position tracking of a body.

→ Angle (θ) → used to identify the position of an object with in the space.

angle $\theta = \frac{\text{arc length}}{\text{(radius)}} = \frac{x}{r} \rightarrow \theta = \frac{x}{r}$

Unit → Radian, degree $\Rightarrow 1^R = \frac{x}{r} \rightarrow x = r$

→ Arc length = radius of the circle

if $\theta = 2^R \rightarrow 2 = \frac{x}{r} \rightarrow x = 2r$

→ Arc = double radius travelled.

1 Rotation (cycle) → $2\pi^R \rightarrow 360^\circ$

half cycle → $\pi^R \rightarrow 180^\circ$

quarter cycle → $\frac{\pi}{2} \rightarrow 90^\circ$

Similarly → $\frac{\pi}{4} \rightarrow 45^\circ$

→ $\frac{\pi}{6} \rightarrow 30^\circ$

→ $\pi/3 \rightarrow 60^\circ$

→ $\frac{2\pi}{3} \rightarrow 120^\circ$, $\frac{3\pi}{2} \rightarrow 270^\circ$

$2\pi^R = 360^\circ \rightarrow 1^R = \frac{360}{2\pi} = \frac{360}{2(3.14)}$

$1^R = 57.3248^\circ \rightarrow 1^R = 57^\circ 19' 29''$

here 1 degree $1^\circ = 60'$ → 60 Minutes.
1 Minute $1' = 60''$ → 60 seconds.
$0.3248^\circ \rightarrow 0.3248 \times 60 \rightarrow 19.488'$
$\rightarrow 19' \; 0.488'$
$0.488' \rightarrow 0.488 \times 60$
$\rightarrow 29.28'' \rightarrow 29''$

Ex: Find angle between needles in wall clock?

According to the scientist Pythagorous in 570-490 BC

we notice that 1) the minimum sides required to form an Area is 3

2) so that, there exist 3 angles, & the total sum is 180°.

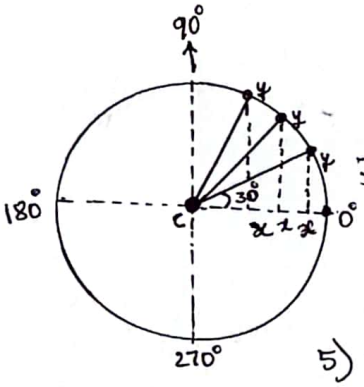
$\alpha + \beta + \gamma = 180^\circ$

if one of the angle is 90° → $\gamma = 90^\circ$ → $\alpha + \beta + 90^\circ = 180^\circ$

$\alpha + \beta = 180 - 90^\circ$

$\alpha + \beta = 90^\circ$

Many triangles → 30+60, 45+45, 20+70 - - - - -



3) The idea of right angled triangle is found in circle.

4) when the particle moving along $0^\circ \div 90^\circ \Rightarrow x \downarrow \; y \uparrow$

$90^\circ - 180^\circ \Rightarrow x \uparrow \; y \downarrow$

$180^\circ - 270^\circ \Rightarrow x \downarrow \; y \uparrow$

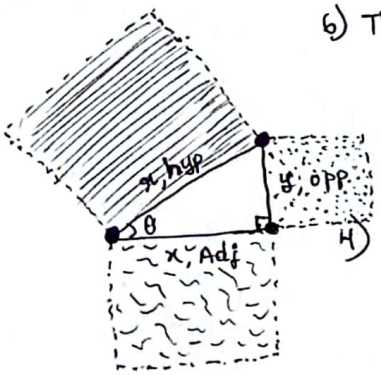
$270^\circ - 360^\circ \Rightarrow x \uparrow \; y \downarrow$

5) Trigonometry is the relation between 3 sides & 3 angles.

6) The value of hypothenuse is resolved into two components

1) weight of a body
2) Electric field intensity
3) velocity of a vehicle
4) Applied Force (push, pull)

horizontal (or) Adjacent side.
vertical (or) opposite side



4) relation between Areas → $hyp^2 = Adj^2 + Opp^2$

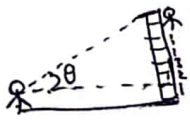
$r^2 = x^2 + y^2$ → circle equation

constant, variables.

on x-axis → $y = 0 \rightarrow x = r$ (sleeping on a bed)

on y-axis → $x = 0 \rightarrow y = r$ (standing, Total weight on a leg)

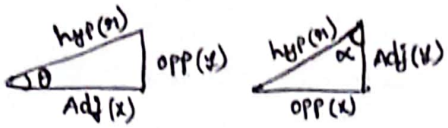
Ex: 1



standing infront of a building to see the person in 10th Floor.

Ex: 2 A Man of weight 50 kg. inclined on a wall at 60°, then his weight can be resolved as 25 kg on his Foot, 43.3 kg on the wall.

Ex: 3 The inclined ladder, moving vehicle on an inclined bridge, weights in simple pendulo



Trigonometric Ratios → Relation between 3 sides

$\cos\theta = \frac{Adj}{hyp} = \frac{x}{r} \longrightarrow x = r\cos\theta$

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} \longrightarrow y = r\sin\theta$

Ex: $\sin 30^\circ = \frac{y}{r} \longrightarrow \frac{1}{2} = \frac{y}{r} \longrightarrow y = \frac{r}{2} \longrightarrow$ opp = 50% of hyp.

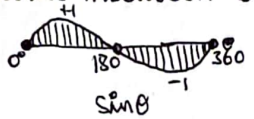
Similarly $\tan\theta = \frac{y}{x}$ → slope , $\cot\theta = \frac{x}{y}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{r}{y}$.

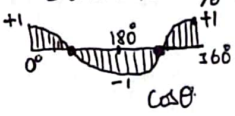
remember ⟹ 0 → 0%, $\frac{1}{2}$ — 50%, $\frac{1}{\sqrt{2}}$ — 70%, $\frac{\sqrt{3}}{2}$ — 86%, 1 → 100%,

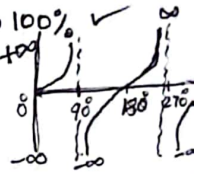
	1st Quadrant					2nd Quadrant				3rd Quadrant				4th Quadrant			
	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$\sin\theta$ (opp side)	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	−1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$\cos\theta$ (Adj)	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	−1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	−1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	−1	$-\frac{1}{\sqrt{3}}$	0

∴ cosθ, sinθ → varies inbetween 0 & 1, that is 0% to 100%

1 Rotation → 360°
2 " → 720°
3 " → 1080°







$-\theta = \infty$ also ∴ sinθ, cosθ → continuous functions, ~~tanθ → dis~~

and tanθ, cotθ, secθ, cosecθ → discontinous functions → having '∞'

12: exponential & logarithmic functions

exponential functions → like a^x, e^x → 2^x, 10^x, e^x here e = 2.718

→ used in population growth, compound interest, Radioactive decay, Heat transfer, $\log_e^x$

→ it is the part of sinθ, cosθ.

$\sin\theta = \frac{e^x - e^{-x}}{2}$, $\cos\theta = \frac{e^x + e^{-x}}{2}$, e^x → natural exponential

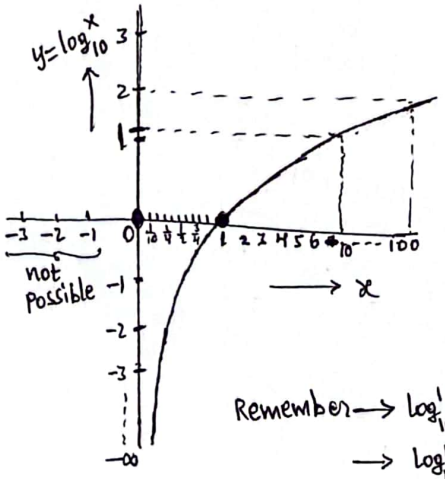
Logarithmic functions → like $\log_e^x$ (or) $\ln_e^x$, $\log_{10}^x$

→ it is the inverse of exponentiation ∴ $a^x = b \rightarrow \log_a^b = x$

→ it compresses large scales into manageable numbers.

$2^x = 8 \rightarrow \log_2^8 = x$
$\rightarrow \log_2^{2^3} = x$
$\rightarrow 3\log_2^2 = x$
$\rightarrow 3(1) = x \Rightarrow x = 3$

→ used in earthquake scale, Astronomy, Sound intensity (decibles), pH value in chemistry.



(1) $\log_{10}^{ab} = \log_{10}^a + \log_{10}^b$	$\ln_e^x = \log_e^{10} \times \log_{10}^x$
(2) $\log_{10}^{a/b} = \log_{10}^a - \log_{10}^b$	$\ln_e^x = 2.3025 \times \log_{10}^x$
(3) $\log_{10}^{a^n} = n\log_{10}^a$	

Remember → $\log_{10}^1 = 0$ ($\because 10^0 = 1$) , $\log_{10}^{1/2} = -0.3010$

→ $\log_{10}^{10} = 1$ ($\because 10^1 = 10$) , $\log_{10}^2 = 0.3010$

→ $\log_{10}^{100} = 2$ ($\because 10^2 = 100$) , $\log_{10}^{0.1} = -1$

→ $\log_{10}^0 = -\infty$ ($\because 10^{-\infty} = \frac{1}{10^\infty} = \frac{1}{\infty} = 0$), $\log_{10}^3 = 0.4771$

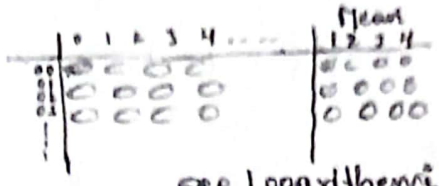
→ $\log_{10}^{-5}$ = error (not possible for ... numbers x), $\log_{10}^4 = 0.602$

we know $10^2 = 100$
$10^3 = 100.0$
then $10^{2.6} = ?$
Ans: it is Antilog (2.6)
⟹ 398.1

See next page for 'log' calculations.

Logarithemic calculations (For laboratory).

→ $\log_{10}^{x}$ = characteristic + mantissa
↓ decimal point
↓ (n-1)
n → no. of digits



see Logarithemic Table.

ex:1 $\log_{10}^{179}$ = char + Mantissa
179 → n = 3, n-1 → 2 ; 17 in 9, 2528 ⟹ $\log_{10}^{179} = 2.2528$.

2) $\log_{10}^{17.9} = 1.2528$, $\log_{10}^{1.79} = 0.2528$

$\log_{10}^{0.179} = \bar{1}.2528$ (0-1) , $\log_{10}^{0.0179} = \bar{2}.2528$.

$a^x = b \rightarrow \log_a^b = x$

$2^3 = 8 \Rightarrow \log_2^8 = 3$

3) $\log_{10}^{2} = 0.3010 \Rightarrow 10^{0.301} = 2$

$\log_{10}^{20} = 1.3010 \Rightarrow 10^{1.301} = 20$, $\log_{10}^{200} = 2.3010$, $\log_{10}^{2000} = 3.3010$.

$\log_{10}^{10} = 1 \Rightarrow 10^1 = 10$, $\log_{10}^{100} = 2 \rightarrow 10^2 = 100$ .....

→ Antilog → let $x = \log_{10}^{y}$ → $10^x = y$ (or) to get 'y' ⟹ $y = \text{Antilog}_{10}^{x}$.

→ Antilog_{10}^{x} = characteristic + Mantissa
(n+1)
n → value of digit.

---

1) $y = 24.5 \times 6.387 = ?$ → $\log_{10}^{y} = \log_{10}^{24.5} + \log_{10}^{6.387}$

$\log_{10}^{y} = 1.3891 + 0.8052 = 2.1943$

$\therefore y = \text{Antilog}(2.1943)$ → (or) $y = 10^{2.1943}$
↓ char → 2+1 = 3 → decimal point after 3rd digit.
Mantissa → see this for after the decimal point.
19 in H under 3 → 1564

$\therefore y = 156.4$ → check in calculator.

2) Area of circle $A = \pi r^2 = 3.14 \times (0.625)^2$

$\log_{10}^{A} = \log 3.14 + \log(0.625)^2 = \log 3.14 + 2\log 0.625$

$\log_{10}^{A} = 0.4969 + 2(\bar{1}.7958) = 0.4969 + \bar{1}.5916 = 0.0885$

$A = \text{Antilog}(0.0885)$ → (or) $A = 10^{0.0885}$
↓ 0+1 = 1st digit. , 08 in 8 under 5

$A = 1.226 \text{ cm}^2$ → check in in calculator.

3) volume of the sphere $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (0.89)^3$

$\log_{10}^{V} = \log_{10}^{4.000} + \log_{10}^{3.000} + \log 3.14 + 3\log(0.89)$.

$\log_{10}^{V} = 0.6020 + 0.4771 + 0.4969 + 3(\bar{1}.9493) = 0.4697$

$V = \text{Antilog}(0.4697) \rightarrow V = 2.949 \text{ cm}^3$

4) volume of cylinder $V = \pi r^2 l = 3.14 \times (0.63)^2 \times 2.56$

$\log_{10}^{V} = \log 3.14 + 2\log 0.63 + \log 2.56$

$= 0.4969 + 2(\bar{1}.7993) + 0.4082 = 0.5037$

$V = \text{Antilog}(0.5037) \rightarrow V = 3.189 \text{ cm}^3$

5) Focal Length $f = \frac{UV}{U+V} = \frac{34 \times 75.3}{34 + 75.3} = \frac{34 \times 75.3}{109.3}$

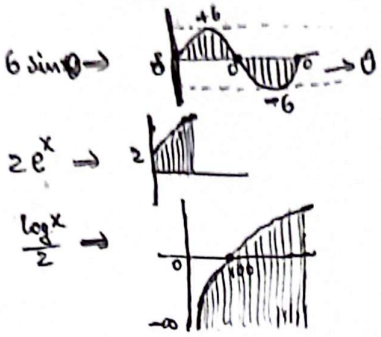
$\log_{10}^{f} = \log 34 + \log 75.3 - \log 109.3 = 1.5314 + 1.8767 - 2.0386$

$\log_{10}^{f} = 1.3695 \rightarrow f = \text{Antilog}(1.3695) = 23.415 \text{ cm}$

Functions, Terms, equations

- **Functions**
  - → it is the part of term
  - → it gives value based on graph
  - → it is x, θ ...
  - → it is on Right side.
- **Terms**
  - → it is the part of equation
  - → it is seperation by $+, -$
  - → it include $x, \div$
  - → having constants along with functions
- **equations**
  - → it is the combination of terms
  - → it gives final answer (output) of a machine
  - → denoted by 'y'.
  - → it is on left side.

Let a machine final output is $y = 6\sin\theta + 4e^x - \frac{\log x}{2}$ → equation



here $6, 4, \frac{1}{2}$ → constants

$\sin\theta, e^x, \log x$ → functions

$6\sin\theta, 2e^x, \frac{\log x}{2}$ → terms ($1^{st}, 2^{nd}, 3^{rd}$ → having $x, \div$)
→ there are seperated by $+, -$

θ, x → input variables → Right side of equation

y → output variable → Final answer → left side of equation

Ex:- 1) Standing infront of a mirror → $\frac{1}{f} = \frac{1}{U} + \frac{1}{V}$ → Mirror → function (constant)
object (U), Image (V) → variable

2) human body health (y) → equation
eye, ear, nose, kidney, liver, heart - - - - → functions.

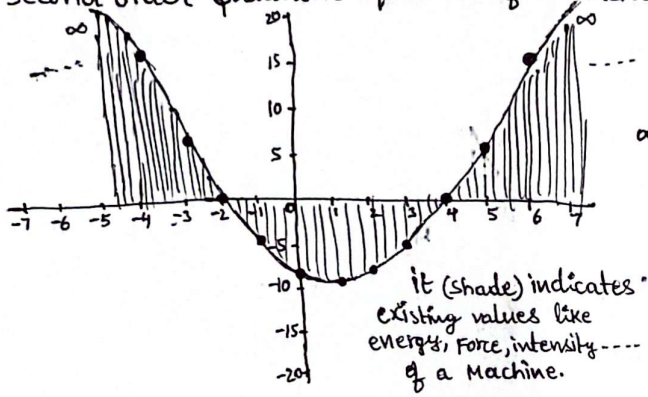
3) computer → equation
Mouse, keyboard, CPU, speeker, Screen - - - → functions.

→ second order quadratic equation of a machine $y = x^2 - 2x - 8$ → 3 terms.

$= x^2 - 4x + 2x - 8$

$= x(x-4) + 2(x-4)$

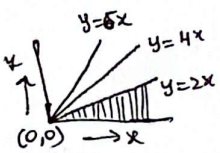
$y = (x-4)(x+2)$.

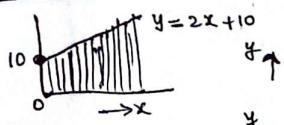


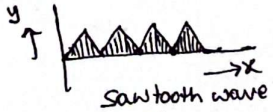
at $x = -4 \rightarrow y = -8 \times -2 = 16$
$-3 \rightarrow y = -7 \times -1 = 7$
$-2 \rightarrow y = -6 \times 0 = 0$
$-1 \rightarrow y = -5 \times 1 = -5$
$0 \rightarrow y = -4 \times 2 = -8$
$1 \rightarrow y = -3 \times 3 = -9$
$2 \rightarrow y = -2 \times 4 = -8$
$3 \rightarrow y = -1 \times 5 = -5$
$4 \rightarrow y = 0$
$5 \rightarrow y = 1 \times 7 = 7$
$6 \rightarrow y = 2 \times 8 = 16$

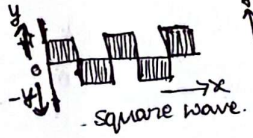
Graphs

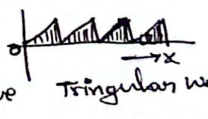
→ straight line:



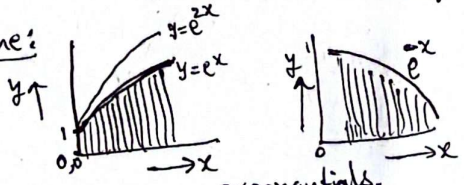


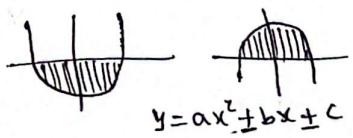




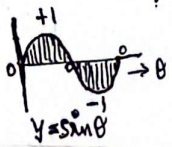


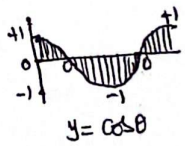
→ curved line:

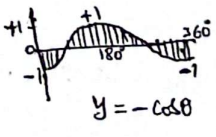




→ Trigonometric:







PROPORTIONALITY THEOREM

Scientists from Ancient times having deep observation on nature like ⟶ when did nature (earth) begins!
⟶ How it was formed!
⟶ How it is running till now?
⟶ what is god?
⟶ what is energy!
⟶ who rules this Universe!

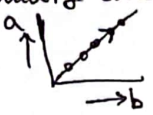
They conclude as: 1) every event in the nature is depends on another event
2) A change in 1st variable is depends on another variable.
3) nothing is constant in the Universe ~~ever~~ except light velocity
4) This dependance is 2 types.

1) $a \propto b$

value of 'a' is directly proportional to value of 'b'.

$\therefore a = kb$ ↳ proportionality constant

if b↑ then a↑
b↓ then a↓

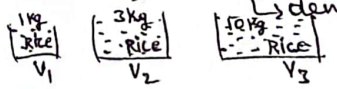


Examples:

1) $a = 10b$
↓
Money to be paid
b = no. of apples required
10 ⟶ 1 apple cost.

if b = 0, 1, 2, 3, 4, - - - - -
a = 0, 10, 20, 30, 40 - - - -

2) mass ∝ volume
$m \propto v$, $m = dv$ ↳ density (constant) (Rice)



3) $V \propto I$, $R \propto l$

4) Force ∝ mass

5) ~~Health~~ Result ∝ Hardwork.

6) work ∝ time

7) ~~Result ∝ B~~
purchase ∝ Money.

2) $a \propto \frac{1}{b}$

value of 'a' is inversely proportional to value of 'b'.

$\therefore a = \frac{k}{b}$ k ⟶ constant

if b↑ then a↓
b↓ then a↑

Examples:

1) $a = \frac{10}{b}$, b = 0, 1, 2, 3, 4, 5 - - - - -

$a = \frac{10}{0}, \frac{10}{1}, \frac{10}{2}, \frac{10}{3}$ - - - -

$= \infty, 10, 5, 3.3, 2.25$ - - -

2) volume ∝ $\frac{1}{\text{density}}$, $V \propto \frac{1}{d}$

$V = \frac{m}{d}$, m ⟶ constant.
let 1 kg ⟶ Rice, Sand, cotton
cotton ⟶ Low density, high volume.

3) $I \propto \frac{1}{R}$, $R \propto \frac{1}{A}$ (Area)

4) Force ∝ $\frac{1}{\text{distance}^2}$

5) pressure ∝ $\frac{1}{\text{Area}}$

6) Health ∝ $\frac{1}{\text{Age}}$

7) Result ∝ $\frac{1}{\text{Laziness}}$

8) Mobile signal ∝ $\frac{1}{\text{Tower distance}}$

This theorem is useful in every subject (every equations).

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