2- VECTORS

Introduction:

We know the some of the physical quantities like distance, time, speed, work, force, pressure, energy, momentum, surface tension, temperature, resistance..........

In above, the magnitudes of the some of the physical quantities are changing with direction. Due to this physical quantities are divided into two types. That is 1)Scalar 2)vector.

1)Scalar:

"A physical quantity which has only magnitude is known as scalar." (OR) A physical quantity which can not changes it's magnitude with change in direction. (OR) A physical quantity which can have equal magnitude in all directions.

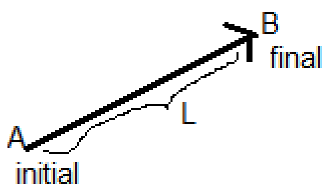
Ex: Time, work, energy, speed, 20kmph, 2, 3x, 6x–9.......

2)Vector:

"A physical quantity which has both magnitude and direction is known as vector." (OR) A physical quantity which can changes it's magnitude with change in direction. (OR) A physical quantity which can have various magnitudes in various directions.

Ex: Velocity, momentum, $2\hat{\imath}$, $3\hat{\jmath} - 6\hat{k}$, 20 kmph in North side.

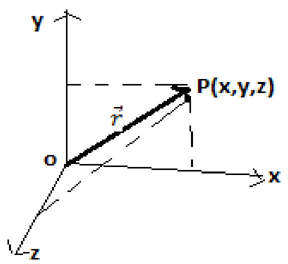
Vector Identification:



A vector can be represented by a direct line segment. It has starting point 'A' and final point 'B'. Arrow indicates the direction and length of the line indicates the magnitude. A vector can be written as $\overrightarrow{AB}$ and magnitude by $|\overrightarrow{AB}|$ (or) simply by AB.

If a vector starts from origin '0' up to some position(point) 'P' then the vector form is $\vec{P}$,and it's magnitude is $|\vec{P}|$ (or) simply P.

Position vector:



In Cartesian coordinate system the position of a particle 'P' is denoted by (x, y, z). Then the position of particle 'P' from the origin is written as $\vec{r} = x\,\vec{\imath} + y\,\vec{\jmath} + z\,\vec{k}$

Magnitude of $\vec{r}$ (or) length of 'P' from origin $|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$

Unit vector: "A vector of unit magnitude is known as unit vector". It can be written as $\hat{A}$ (or) $\hat{r}$ (or) $\hat{n}$

Unit vector of vector $\vec{A}$ is $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$, and magnitude of unit vector is $|\hat{A}| = \frac{|\vec{A}|}{|\vec{A}|} = 1$

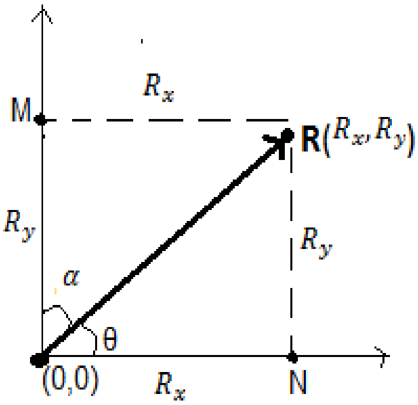
$\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are the unit vectors along x, y, z axes. Then $|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$

Ex: if vector $\vec{A} = 2\,\vec{\imath} - 3\,\vec{\jmath} + 4\,\vec{k}$ then unit vector is $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\,\vec{\imath} - 3\,\vec{\jmath} + 4\,\vec{k}}{\sqrt{2^2+(-3)^2+4^2}} = \frac{2\,\vec{\imath} - 3\,\vec{\jmath} + 4\,\vec{k}}{\sqrt{29}} = \frac{2}{\sqrt{29}}\hat{\imath} - \frac{3}{\sqrt{29}}\hat{\jmath} + \frac{4}{\sqrt{29}}\hat{k}$

Null vector: "A vector of zero magnitude is known as null vector".

Ex: Velocity of a body at maximum height when it thrown up vertically.

------->Resolution of a vector (or) component of a vector in a plane: (Two dimension)



Let us take a vector $\vec{R}$ in xy plane. The perpendiculars of $\vec{R}$ on x and y axes are N,M. The angle taken by the $\vec{R}$ with x-axis is 'θ'.

The component of $\vec{R}$ along x-direction is R_x ----->Horizontal component.
The component of $\vec{R}$ along y-direction is R_y ----->Vertical component.

Then the vector can be written as $\vec{R} = R_x\,\hat{\imath} + R_y\,\hat{\jmath}$

From Δ OAN -----> $\cos\theta = \frac{ON}{OA} = \frac{R_X}{R}$ -------> $R_x = R\cos\theta$

----> $\sin\theta = \frac{AN}{OA} = \frac{R_y}{R}$ -------> $R_y = R\sin\theta$

There fore $\vec{R} = R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}$

Now $R_x^2 + R_y^2 = (R\,cos\theta)^2 + (R\,sin\theta)^2 = R^2(1)$

$R_x^2 + R_y^2 = R^2$ --------> $\boxed{R = \sqrt{R_x^2 + R_y^2}}$

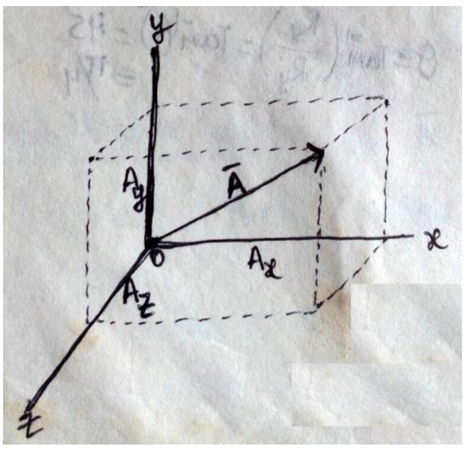
Therefore any vector $\vec{R} = R\,\hat{R}$, angle 'θ' can be taken as $\tan\theta = \frac{AN}{ON} = \frac{R_y}{R_x}$ ------> $\boxed{\theta = \tan^{-1}(\frac{R_y}{R_x})}$

Note: 1)if R_x= 0------> R = R_y , vector lies on y-axis, angle θ = 90°

2)if R_y= 0------> R = R_x , vector lies on x-axis, angle θ = 0°

3)if $R_x = R_y$------> R = $\sqrt{R_x^2 + R_y^2} = \sqrt{2}\,R_x$ (or) $\sqrt{2}\,R_y$, vector lies in middle of xy plane, angle θ = 45°

Addition of vectors by components method:



Any vector $\vec{A}$ in three dimensional coordinate system can be represented by $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$

Here A_x , A_y , A_z are the scalar components of $\vec{A}$ along x, y, z axes(directions) and $A_x\,\hat{\imath}$, $A_y\,\hat{\jmath}$, $A_z\,\hat{k}$ are the vector components of $\vec{A}$ along x, y ,z directions.

If there is another vector $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$ in three dimensional coordinate system, then

It's addition is $\vec{A} + \vec{B} = (A_x+B_x)\hat{\imath} + (A_y+B_y)\hat{\jmath} + (A_z+B_z)\hat{k}$
and subtraction is $\vec{A} - \vec{B} = (A_x-B_x)\hat{\imath} + (A_y-B_y)\hat{\jmath} + (A_z-B_z)\hat{k}$

Problem: if $\vec{A} = 2\,\hat{\imath} + 5\,\hat{\jmath} + 4\,\hat{k}$ and $\vec{B} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k}$ find $|\vec{A}|$, $|\vec{B}|$, $\hat{A}$, $\hat{B}$, $|\vec{A} + \vec{B}|$, $|\vec{A} - \vec{B}|$, $|\vec{B} - \vec{A}|$

Addition of vectors:

If the given physical quantities are correspond to same type then only we can add the vectors.

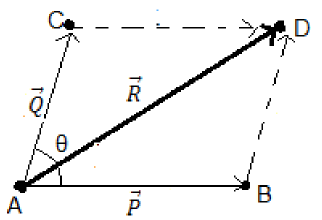
Addition of vectors can be done in 3 ways. 1)by parallelogram law of vectors
2)by Triangle law of vectors
3)by Polygon law of vectors

------------>State parallelogram law of vectors? Derive an expression for resultant magnitude and direction of two vectors?

Parallelogram law of vectors:

"If two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, then the diagonal passing through that point represents their resultant vector both in magnitude and direction."

Explanation:

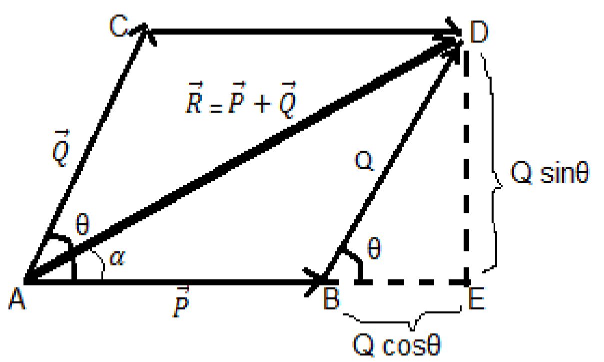


Let $\vec{P}$ *and* $\vec{Q}$ represents the adjacent sides $\overrightarrow{AB}$ *and* $\overrightarrow{AC}$ of a parallelogram, and 'θ' is the angle between them.

If parallelogram ABCD is completed then diagonal $\overrightarrow{AD}$ indicates the resultant vector $\vec{R} = \vec{P} + \vec{Q}$

Magnitude of resultant vector:

To find the magnitude of the resultant vector $\vec{R}$, we can extend the line AB up to E and draw a perpendicular line from D on E.



Then ∟CAB = ∟DBE = θ and BE = Q cos θ, DE = Q sin θ

$|\overrightarrow{AB}|$ = P , $|\overrightarrow{AC}|$ = Q

From Δ DAE , by applying Pythagoras theorem,

We have $AD^2 = AE^2 + DE^2$

$= (AB + BE)^2 + DE^2$

$= (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$= P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$

$R^2 = P^2 + 2PQ\cos\theta + Q^2$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

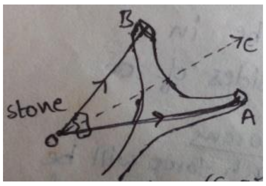
Let resultant $\vec{R}$ makes an angle 'α' with $\vec{P}$ then from Δ DAE , $\tan\alpha = \frac{DE}{AE} = \frac{DE}{AB+BE}$

$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$ ----> $\boxed{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)}$

---->Examples of parallelogram law of vectors:

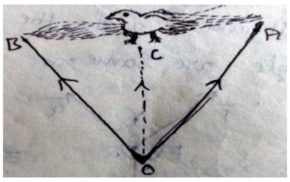
1)Working of catapult 2)Bird flying with wings 3)Moving arrow 4)Motion of a boat in a river

----->Explain the examples of parallelogram law of vectors?



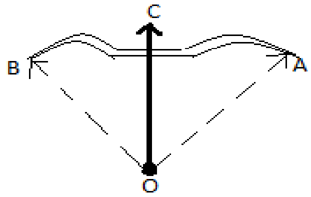
1)Working of catapult:

In this resultant forces acts in the direction OC on the stone due to the restoring forces along the elastic threads with directions OA and OB.



2)Bird flying with wings:

When bird pushes air in AO and BO directions, the reactions of air acts on the wings in the directions OA and OB, hence bird flies in OC direction.



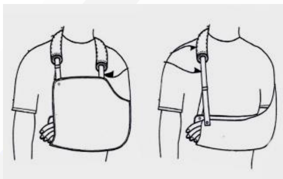
3)Motion of arrow:

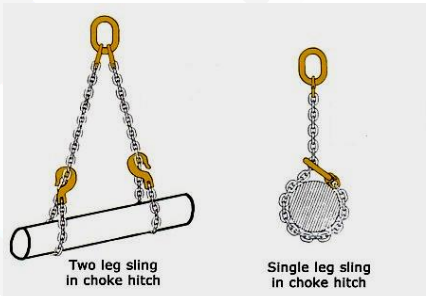
When arrow pulled back due to the restoring forces along OA and OB then arrow moves in OC direction.

4)Sling:

A flexible strap (or) belt in the form of a loop to support (or) raise a hanging weight.

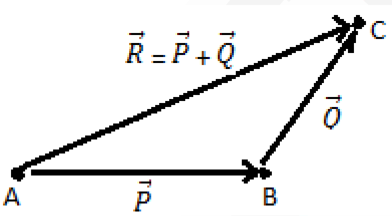
Ex: a baby had to be supported by a sling fixed to the Mother's shoulder.





------->Explain the Triangle law of vectors?

Statement: "If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, then their resultant represented in magnitude and direction by the third side of the triangle taken in reverse order."

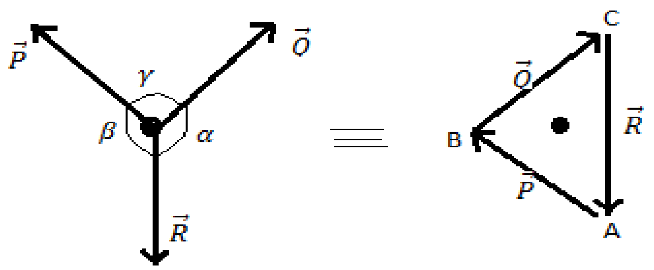


Consider two vectors $\overrightarrow{AB} = \vec{P}$ and $\overrightarrow{BC} = \vec{Q}$ of the triangle ABC, then the resultant of $\vec{P}$ and $\vec{Q}$ of the same triangle taken in opposite order. Here ending position of $\vec{P}$ is equal to initial position of $\vec{Q}$.

That is $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ ------> $\vec{R} = \vec{P} + \vec{Q}$

Another statement of triangle law:

If three forces acting on a body can be represented in magnitude and direction by the three sides of a triangle taken in order, then the body will be in equilibrium , and the ratios of forces to the sides of a triangle are same.



Consider three forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting on body at a point '0' . Due to this make a triangle ABC with sides AB, BC, CA.

The resultant force = The sum of three forces

$$= \vec{P} + \vec{Q} + \vec{R}$$

$$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$$

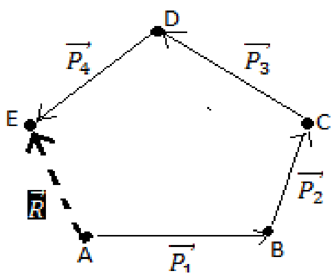
$$= \overrightarrow{AC} + \overrightarrow{CA}$$

Resultant force $= \overrightarrow{AC} - \overrightarrow{AC} = 0$

That is body is in equilibrium.

-------->Explain the Polygon law of vectors?

Statement: "If a number of vectors are represented in magnitude and direction by the sides o a polygon taken in order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in reverse order."



Consider the vectors $\overrightarrow{P_1}$, $\overrightarrow{P_2}$, $\overrightarrow{P_3}$, $\overrightarrow{P_4}$ are represented in magnitude and direction by the AB, BC, CD and DE.

By applying the triangle law in step by step, we can find resultant vector $\vec{R}$.

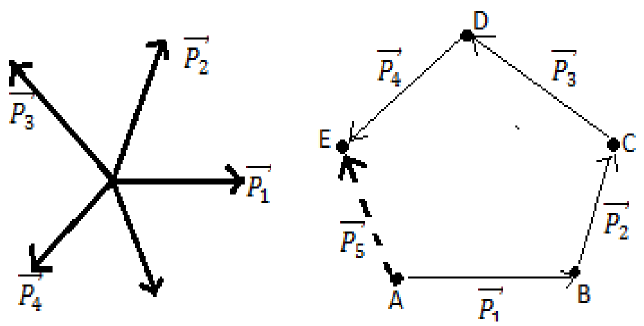
That is $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{P_1} + \overrightarrow{P_2}$

$$\overrightarrow{AD} = \overrightarrow{AD} + \overrightarrow{CD} = \overrightarrow{P_1} + \overrightarrow{P_2} + \overrightarrow{P_3}$$

There fore $\vec{R} = \overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE}$ -----> $\vec{R} = \overrightarrow{P_1} + \overrightarrow{P_2} + \overrightarrow{P_3} + \overrightarrow{P_4}$

Another Statement:

If a number of vectors acting at a point are represented in magnitude and direction by the sides of a closed polygon taken in order, then the point will be in equilibrium.



Here the sides AB, BC, CD, DE and EA represents the forces $\overrightarrow{P_1}$, $\overrightarrow{P_2}$, $\overrightarrow{P_3}$, $\overrightarrow{P_4}$, $\overrightarrow{P_5}$ respectively.

Then the resultant force = sum f all the forces.

$$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$$

Resultant force $= \overrightarrow{AE} + \overrightarrow{EA} = \overrightarrow{AE} - \overrightarrow{AE} = 0$

That is point (or) body is in equilibrium.

------>Explain the subtraction of vectors with diagrams?

Statement:

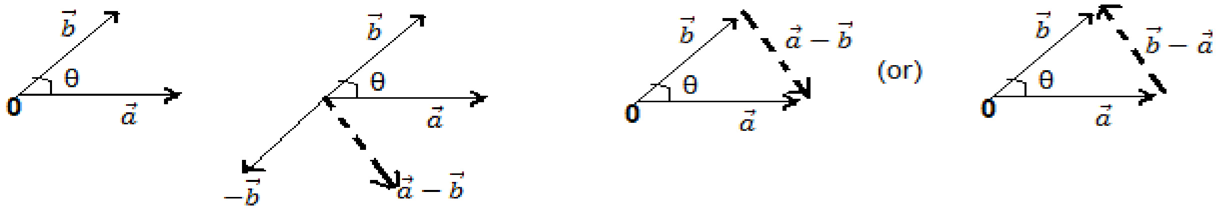
" If two vectors starting from a point '0' with an angle 'θ' then the closing side represents the $\vec{a} - \vec{b}$."

Step(1): consider two vectors $\vec{a}$ and $\vec{b}$ with a angle 'θ'.

Step(2): Take opposite vector to $\vec{b}$ that is $-\vec{b}$.

Step(3): Applying parallelogram law , we get resultant vector $\vec{a} - \vec{b}$.

Step(4): Take parallel vector at ending positions of given vectors, it will shows resultant subtraction.



Multiplication of vectors: Vector multiplication can be done in 3 ways.

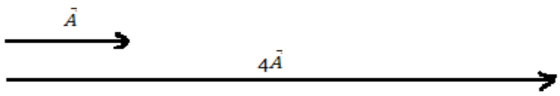
1)Multiplication of vector by scalar, result is vector.
2) Multiplication of vector by vector, result is scalar.
3) Multiplication of vector by vector, result is vector.

------->(1) Multiplication of vector by scalar, result is vector:

Consider a scalar 'k' and a vector $\vec{A}$, then multiplication of vector $\vec{A}$ with scalar 'k' represents a vector. This resultant vector gives the k times greater than of vector $\vec{A}$. It is represented by k$\vec{A}$.

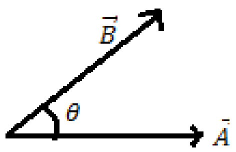
Ex: if vector $\vec{A} = 2\,\hat{\imath} - 3\,\hat{\jmath} + 4\,\hat{k}$ and k = 4 (scalar)

Then $k\vec{A} = 4\vec{A} = 4\,(2\,\hat{\imath} - 3\,\hat{\jmath} + 4\,\hat{k}\,) = 8\,\hat{\imath} - 12\,\hat{\jmath} + 16\,\hat{k}$



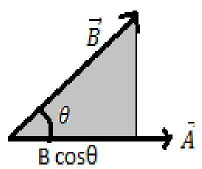
--------->(2)DOT product (or) Scalar product of two vectors:

If the product of two vectors results scalar then the product is known as Dot (or) scalar product.

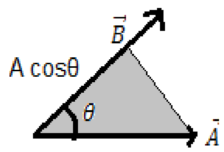


Definition: The dot product of two non zero vectors is defined as the product of their magnitudes multiplied by cosine of the angle between them.

Let $\vec{A}$, $\vec{B}$ are two non zero vectors then the scalar (or) DOT product is



$\vec{A} \bullet \vec{B} = |\vec{A}|\ |\vec{B}|\cos\theta = A\,B\cos\theta$ = scalar



$\vec{A} \bullet \vec{B}$ = A (B cos θ) = (magnitude of $\vec{A}$) x (component of Let $\vec{B}$ on $\vec{A}$)

$\vec{A} \bullet \vec{B}$ = (A cos θ) B = (component of Let $\vec{A}$ on $\vec{B}$) x (magnitude of $\vec{B}$)

Properties of DOT product:

1)Scalar product obeys commutative law. That is $\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$

2)Scalar product does not obeys Associative law. That is $(\vec{A} \bullet \vec{B}) \bullet \vec{C} \neq \vec{A} \bullet (\vec{B} \bullet \vec{C})$

3) Scalar product obeys Distributive law. That is $\vec{A} \bullet (\vec{B} + \vec{C}) = (\vec{A} \bullet \vec{B}) + (\vec{A} \bullet \vec{C})$

4)If $\vec{A}$, $\vec{B}$ are parallel to each other then DOT product is maximum.

That is $\theta = 0^o$ -----> $\vec{A} \bullet \vec{B} =$ A B $\cos 0^o$ = AB (1) = AB = max = $\sqrt{A_x^2 + A_y^2 + A_z^2}$ x $\sqrt{B_x^2 + B_y^2 + B_z^2}$

5)If $\vec{A}$, $\vec{B}$ are perpendicular to each other then DOT product becomes Zero.

That is $\theta = 90^o$ -----> $\vec{A} \bullet \vec{B} =$ A B $\cos 90^o$ = AB (0) = 0

6)Since the unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are perpendicular to each other, then $\hat{\imath} \bullet \hat{\imath} = 1$, $\hat{\jmath} \bullet \hat{\jmath} = 1$, $\hat{k} \bullet \hat{k} = 1$
and $\hat{\imath} \bullet \hat{\jmath} = 0$, $\hat{\jmath} \bullet \hat{k} = 0$, $\hat{k} \bullet \hat{\imath} = 0$

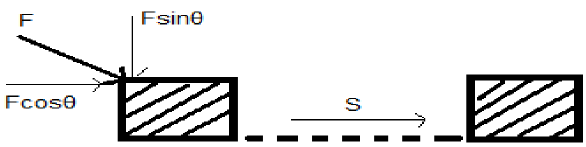
7)Angle between any two vectors is

$$\vec{A} \bullet \vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta = \text{A B} \cos\theta \text{ -------------->} \ \theta = \cos^{-1}\left[\frac{\vec{A} \bullet \vec{B}}{|\vec{A}|\ |\vec{B}|}\right]$$

8)In Cartesian coordinate system the DOT product of two vectors $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$, $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$
is $\vec{A} \bullet \vec{B} = A_xA_x + B_yB_y + B_zB_z$ = SCALAR ----> no directions

Examples of Scalar product:

a)Work done by a force:



In general, work is known as the DOT product of force and displacement.

Work is a scalar quantity.

Work W = $\vec{F} \bullet \vec{S}$ = F (S $\cos\theta$) = magnitude of force x component of displacement in force direction.

W = $\vec{F} \bullet \vec{S}$ = (F $\cos\theta$) S = component of force in displacement direction x magnitude of displacement.

b)Power:

"Power is equal to the rate of doing work."

P = $\frac{w}{t} = \frac{\vec{F} \bullet \vec{S}}{t} = \vec{F} \bullet \vec{V}$ = DOT product of force and velocity, Power is a scalar quantity.

c)Potential energy:

The work done in lifting a body of mass 'm' to a height 'h' by applying a vertical force F will be stored in it as P.E = W = $\vec{F} \bullet \vec{S}$ = FS $\cos 90^0$ = m(–g) h (1) = –mgh

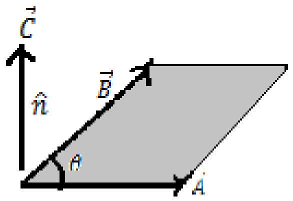
d)Magnetic flux: ϕ = $\vec{B} \bullet \vec{A}$, Here $\vec{B}$ is magnetic flux density (or) induction vector
$\vec{A}$ is surface area vector.

------>(3)CROSS product (or) vector product of two vectors:

If the product of two vectors results a vector then the product is known as vector product.

Definition:

The CROSS product of two non zero vectors is defined as the product of their magnitudes multiplied by sine of the angle between them.



The resultant of the product is also a vector and it's direction is normal to the plane containing the two vectors.

Let $\vec{A}$, $\vec{B}$ are two non zero vectors then the vector (or) CROSS product is

$\vec{C} = \vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta\, \hat{n} = A\,B \sin\theta\, \hat{n}$ = vector , here $\hat{n}$ is resultant direction.

-------> Properties of CROSS product:

1)Vector product does not obeys commutative law. That is $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ and $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

2)Vector product does not obeys Associative law. That is $(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$

$$(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{A} \bullet \vec{B})\,\vec{C} - (\vec{A} \bullet \vec{C})\,\vec{B}$$

3)Vector product obeys Distributive law. That is $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$

4)If $\vec{A}$, $\vec{B}$ are parallel to each other then CROSS product is Zero.

That is $\theta = 0^o$ -----> $\vec{A} \times \vec{B} = A\,B \sin 0^o = AB\,(0) = 0$

5)If $\vec{A}$, $\vec{B}$ are perpendicular to each other then CROSS product becomes Maximum.

That is $\theta = 90^o$ -----> $\vec{A} \times \vec{B} = A\,B \sin 90^o\, \hat{n} = AB(1)\,\hat{n} = AB = \sqrt{A_x^2 + A_y^2 + A_z^2} \times \sqrt{B_x^2 + B_y^2 + B_z^2}\ \hat{n}$

6)Since the unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are perpendicular to each other, then $\hat{\imath} \times \hat{\imath} = 0$, $\hat{\jmath} \times \hat{\jmath} = 0$, $\hat{k} \times \hat{k} = 0$

and $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$ and also $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, $\hat{k} \times \hat{\jmath} = -\hat{\imath}$, $\hat{\imath} \times \hat{k} = -\hat{\jmath}$

8)In Cartesian coordinate system the CROSS product of two vectors $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$, $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$

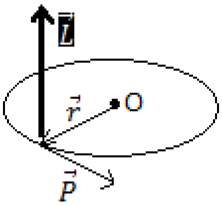
is $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_z\,B_y)\,\hat{\imath} - (A_x\,B_z - A_z\,B_x)\,\hat{\jmath} + (A_x\,B_y - A_y\,B_x)\,\hat{k}$$

9) $|\vec{A} \times \vec{B}|$ gives the area of the parallelogram formed by two vectors. $\frac{|\vec{A} \times \vec{B}|}{2}$ gives triangle area.

10)Volume of the parallelepiped is $[\vec{A}, \vec{B}, \vec{C}] = \vec{A} \bullet (\vec{B} \times \vec{C})$ = SCALAR

-------->Examples of vector product:

a)Angular momentum:



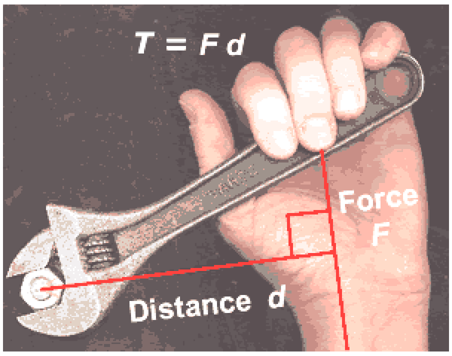
When a particle ‘A’ of mass ‘m’ in circular motion with radius vector $\vec{r}$ and linear momentum $\vec{P}$ then it has angular momentum which perpendicular to both $\vec{P}$ and $\vec{r}$, and it is the cross product of $\vec{r}$ and $\vec{P}$.

There fore Angular momentum $\vec{L} = \vec{r} \times \vec{P}$ ----->Anti clock wise

$\vec{L} = -\vec{P} \times \vec{r}$ ------> Clock wise

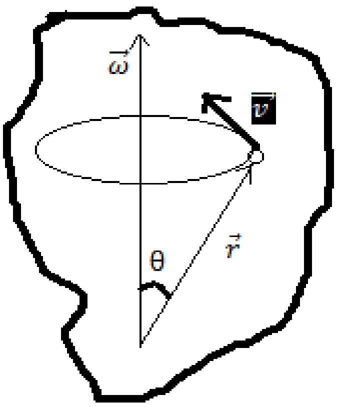
b)Moment of force (or) Torque :

if a force $\vec{F}$ acts on a particle of position $\vec{r}$ with an angle ‘θ’ then the moment of force (or) Torque on the particle is the cross product of $\vec{r}$ and $\vec{F}$.



That is $\tau = \vec{r} \times \vec{F} = r\,F \sin\theta\, \hat{n}$, here $\hat{n}$ -----> $\hat{\imath}$ (or) $\hat{\jmath}$ (or) $\hat{k}$

c)Linear Velocity:



Consider a body rotating about an axis ‘oy’ with angular velocity $\vec{\omega}$. Consider a particle ‘P’ of the body at a distance r $\sin\theta$ from axis of rotation (oy) and position $\vec{r}$ from the origin.

Then the linear velocity $\vec{v}$ is the cross product $\vec{r}$ and $\vec{r}$.

That is $\vec{v} = \vec{r} \times \vec{\omega} = r\,\omega \sin\theta\, \hat{n}$

This linear velocity $\vec{v}$ of the particle of any instant will be perpendicular to the plane containing $\vec{r}$ and $\vec{\omega}$.

PROBLEMS:

1)Find the DOT product and CROSS product of two vectors $\vec{A} = 2\,\vec{i} + 3\,\vec{j} + 4\,\vec{k}$ and $\vec{B} = 4\,\vec{i} - 2\,\vec{j} + 3\,\vec{k}$
-------**Ans:** 14, and $17\,\vec{i} + 10\,\vec{j} - 16\,\vec{k}$

2)Two forces 30N and 40N acts on a body perpendicularly, find resultant force in magnitude and direction? -----2 times **Ans**: R=50N, $\alpha = \tan^{-1}(\frac{40}{30}) = 53.13^o$

3)A force $4\,\vec{i} + 3\,\vec{j} + 6\,\vec{k}$ acts on a body and produces displacement $3\,\vec{i} + 2\,\vec{j} + 5\,\vec{k}$.
Calculate work done? ------2 times **Ans**: W = 48 joules

4)Find the angle between two forces 1N and 24N which produces a resultant of 25N. -------- **Ans**: $\theta=0^0$

5)Two vectors $\vec{A} = \vec{i} + 2\,\vec{j} + n\,\vec{k}$ and $\vec{B} = 4\,\vec{i} + 2\,\vec{j} - 2\,\vec{k}$ are perpendicular vectors, find the value of n.-----**Ans:** n=4

6)A force of 200N is inclined at an angle 30^o to the horizontal. find the vertical and horizontal components.
Ans: vertical ----100 N, Horizontal------173.2 N

7) A force $(2\,\vec{i} + 3\,\vec{j} + 5\,\overrightarrow{k)}$ N acts on a body and produces position vector $(3\,\vec{i} + 12\,\vec{j} + 6\,\overrightarrow{k)}$ m. Calculate the torque produced by the force. **Ans**: $-42\,\vec{i} + 3\,\vec{j} + 15\,\vec{k}$

8)A force of $7\,\vec{i} + 13\,\vec{j} + 9\,\vec{k}$ produces a displacement $2\,\vec{i} + 3\,\vec{j} + 5\,\vec{k}$. Find work done. **Ans**: Work W = 98 joule

9)A force of $\vec{i} + 2\,\vec{j} + 3\,\vec{k}$ produces a displacement $2\,\vec{i} + 3\,\vec{j} + 4\,\vec{k}$. Find work done. **Ans**: Work W = 20 joule

10)force 20N acts on a body with an angle $\theta=60^0$ and produces displacement 10m. Find work? **Ans**: W=100joule

11)Find the angle between two vectors $\vec{A} = 2\vec{i} + 2\,\vec{j} + 4\,\vec{k}$ and $\vec{B} = 3\,\vec{i} - \vec{j} + 2\,\vec{k}$. **Ans:** $\theta=\cos^{-1}(\sqrt{\frac{3}{7}})$

Extra Knowledge problems:

12) if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ then angle between $\vec{A}\ and\ \vec{B}$ is. Ans: $\theta=90^0$

13) if $|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$ then angle between $\vec{A}\ and\ \vec{B}$ is. Ans: $\theta=45^0$

14) Find the value of $|\vec{A} + \vec{B}|^2 + |\vec{A} - \vec{B}|^2$ = ________ Ans: $2(A^2 + B^2)$

15) Find the value of $|\vec{A} + \vec{B}|^2 - |\vec{A} - \vec{B}|^2$ = ________ Ans: 4ABcosθ

16) Find the value of $|\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2$ = ________ Ans: A^2B^2

17) $\vec{A} \cdot \vec{B} = \frac{\sqrt{3}}{2}$ AB then what is the angle between $\vec{A}\ and\ \vec{B}$ is Ans: $\theta=30^0$

18) If two equal forces acts on a body, the resultant force is also equal to either force then what is the angle between two forces(vectors)? (Hint : take P=Q=R) Ans: $\theta=120^0$

19)Two forces $F_1\ and\ F_2$ of magnitudes in the ratio 3:5 acts with an angle $\theta=60^0$ then the resultant force Is 35 N then find individual forces. Ans: F_1 = 15 N , F_1 = 25 N

20)The magnitudes of Scalar product and Vector product of two vectors are $48\sqrt{3}$ and 144 then find the angle between the two vectors? Ans: $\theta=60^0$