3)DYNAMICS

Introduction:

Kinematics is the part of dynamics dealing without reference to force (or) cause of motion. In this we can study the physical quantities like time, speed, velocity, distance, displacement, acceleration, de acceleration and their relations. Kinetics is the part of dynamics dealing with reference of force (or) cause of motion- that is about mass, weight, momentum, force, work, power, pressure, impulse.

Motion: When the body changes it's position relative to another body then the body is said to be in motion. Motion is three types. They are Translation motion , Rotational motion, Vibration motion.

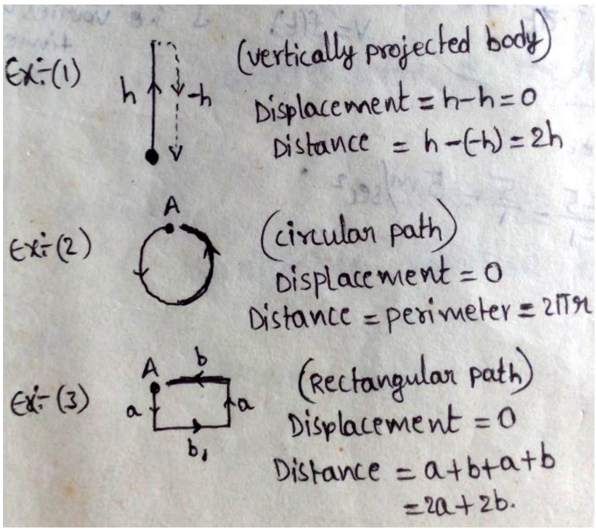
Translation motion-----Motion is in straight line (or) one way (or) only forward motion. Ex: Journey

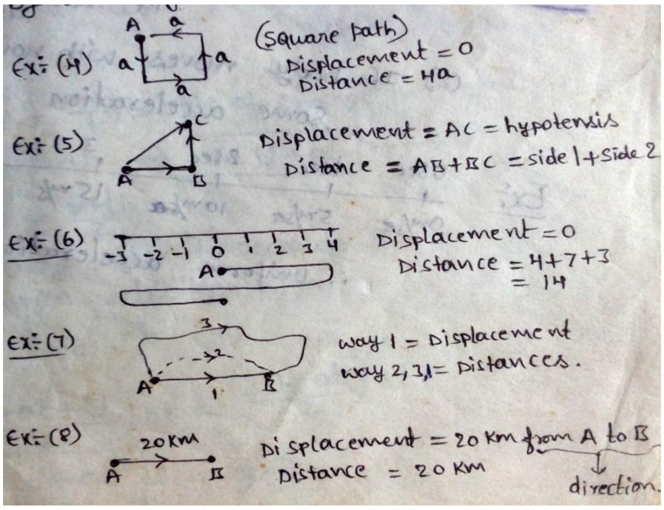
Rotational motion------Motion is along a fixed axis. Ex: Spinning top, earth spin and revolution, electron in orbit

Vibration motion------Motion is about a fixed point and to and fro. Ex: SHM, pendulum, cell phone vibration.

Displacement: The change in position of a body is known as displacement (or) the minimum length (or) distance in between initial and final positions of a body is known as displacement. It is a vector quantity.

Distance: The total length covered by a body due to it's entire motion. It is a scalar quantity.





Time: The experience covered in between initial and final situations of an event is known as time. It is a Scalar quantity. It is a "fourth dimension" used to measure a physical change.

Velocity: It is the rate of change of position of a body along a particular direction. It is a scalar quantity.
(or) The rate at which an object changes it's position.

$$V = \frac{s}{t} \quad \text{(or)} \quad V = \frac{ds}{dt} = \frac{S_2 - S_1}{t_2 - t_1} = \frac{S_2 - S_1}{t - 0} = \frac{S_2 - S_1}{t} \text{------->} S_2 = S_1 + V\,t, \quad \text{V---units---metre /sec---} M^0 L^1 T^{-1}$$

Speed: It is the rate of change of a body along any path (or) curve. It is a scalar quantity (or) It is the magnitude of velocity (or) How fast an object is moving is known as speed. $V = \frac{d}{t}$ V---units---metre /sec---$M^0 L^1 T^{-1}$

Uniform velocity: If a body travels equal displacements in equal interval of times then it has uniform velocity.

Ex: Moving car with 50km/hour . no increasing and no decreasing.

Non uniform velocity: If a body moves with unequal displacements in equal intervals of time.

Ex: moving car with 20 $\frac{km}{h}$ in 1st second, 32$\frac{km}{h}$ in 2nd second, 39$\frac{km}{h}$ in 3rd second, 18$\frac{km}{h}$ in 4th second---

Acceleration: The rate of change of velocity of a body is known as acceleration. It is a vector quantity.

a = v/t (or) $a = \frac{dv}{dt} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{V-U}{t-0} = \frac{V-U}{t}$-------> v= u + a t , a---units---metre /sec^2---$M^0 L^1 T^{-2}$

If a body moves with non uniform velocity(v≠u) means there is some acceleration. $a = \frac{dv}{dt} = \frac{V-U}{t} \neq 0$, a>0 or a<0

Non uniform acceleration: If the rate of change of velocity of the body is same throughout the motion then the acceleration of the body is known as uniform acceleration.

If a body moves with uniform velocity(v=u) means there is no any acceleration--->$a = \frac{dv}{dt} = \frac{V-U}{t} = 0$

Ex: 0 sec................1 sec................2 sec.................3 sec..................4 sec................5 sec........
0 m/s 5m/s 10m/s 15m/s 20m/s 25m/s

Uniform acceleration $a = \frac{(10-5)}{(2-1)} = \frac{5}{1}$ =5 m/sec^2

Acceleration due to gravity:

" The acceleration, which a body experiences while falling freely on to the earth under gravity is known as acceleration due to gravity. It is denoted by 'g'.

At the poles of the earth g = 9.83 m/sec^2 -------> Max
At the equator on the earth g = 9.78 m/sec^2 ----->Min
Therefore the average value of 'g' for the earth is g = 9.805m/sec^2

Value of 'g' slightly depending on the factors like depth, altitude, latitude and rotation of earth. It is Zero at centre of the earth (h=R=6400km).

On the surface of the moon it's value is decremented by '6'. That is $g_{moon} = \frac{g_{earth}}{6} = \frac{9.8}{6} =$ 1.64 m/sec^2

Equations of motion under uniform acceleration: (in a straight line)

If a body moving along a straight line with uniform velocity v then the displacement of the body after 't' seconds is S = V x t.

But if the body moving with initial velocity 'U' along a curved path with uniform acceleration 'a' and final velocity 'V' then we have below equations of motion.

1) V = U + at ------->to find final velocity after t seconds
here U=initial velocity, V=final velocity, a=acceleration, t=time, S=displacement

2) $S = Ut + \frac{1}{2} at^2$ ------->to find distance travelled after 't' seconds

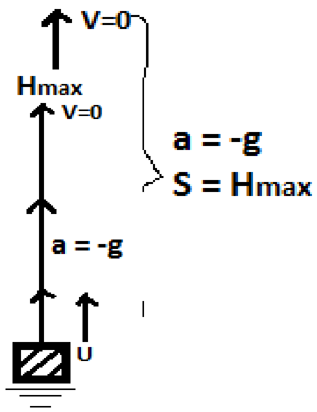
3) $V^2 - U^2 = 2aS$ -------->relation without time

4) $S_n = U + a (n - \frac{1}{2})$ -------> S_n=distance travelled in nth second.---->m/sec

Ex: : 0 sec................1 sec................2 sec.................3 sec..................4 sec................5 sec........
0 m 25m 40m 63m 99m 150m----> $S = Ut + \frac{1}{2} at^2$
<--------25---------><--------15------><--------23-------><--------36--------><-------51-------> → $S_n = U + a (n - \frac{1}{2})$

------>Equations of motion of a body for a freely falling body:

Equations of motion of a uniform accelerated body are



1) V = U + at

2) $S = Ut + \frac{1}{2}at^2$

3) $V^2 - U^2 = 2aS$

4) $S_n = U + a\,(n - \frac{1}{2})$

If a body comes down freely from some height then it has an acceleration a = **g**. In this situation the equations of motion of a body is

1) V = U + **g**t

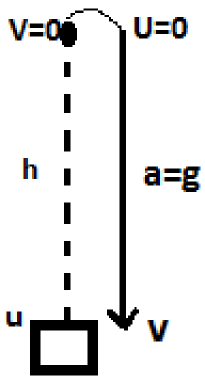
2) $S = Ut + \frac{1}{2}\mathbf{g}t^2$

3) $V^2 - U^2 = 2\mathbf{g}S$

4) $S_n = U + \mathbf{g}\,(n - \frac{1}{2})$ ------> motion towards the ground.

-------->Equations of motion for a vertically thrown up body:

Equations of motion of a uniform accelerated body are



1) V = U + at

2) $S = Ut + \frac{1}{2}at^2$

3) $V^2 - U^2 = 2aS$

4) $S_n = U + a\,(n - \frac{1}{2})$

When a body is thrown up vertically it experiences de acceleration a = – **g**. In this situation the equations of motion of a body is

1) V = U – **g**t

2) $S = Ut - \frac{1}{2}\mathbf{g}t^2$

3) $V^2 - U^2 = -2\mathbf{g}S$

4) $S_n = U - \mathbf{g}\,(n - \frac{1}{2})$ ------> motion opposite to the ground.

------>Derive an expressions for parameters of vertically thrown up body? (or) derive the expressions for Maximum height, time of ascent, time of descent, time of flight for a body thrown up vertically?

Maximum height reached by the body:

It is the maximum distance travelled by a vertically thrown up body before its velocity becomes Zero. If a body thrown up vertically with a uniform velocity 'U' then the body acquire de acceleration a = –**g**.

When the body reaches maximum height it's final velocity becomes Zero, that is V=0.

Then from equation $V^2 - U^2 = 2aS$, we get $0^2 - U^2 = -2\mathbf{g}H_{max}$ -----> $\boxed{H_{max} = \frac{U^2}{2\mathbf{g}}}$

And initial velocity $U = \sqrt{2gH}$ -----> $U \propto \sqrt{H}$

Time of ascent:

The time taken by the body to reach the maximum height is called time of ascent. When a body thrown up vertically with initial velocity 'U' with acceleration a = –**g**.

At a maximum height the final velocity V=0.

Substitute V = 0, a = –**g**, t = t_a in V = U + at

we get $0 = U - \mathbf{g}\,t_a$ -----> $\boxed{t_a = \frac{U}{g}}$

Time of Descent:

The time taken by the body to reach the point of projection from maximum height is known as time of Descent.

When a body falling from maximum height $S = H_{max}$ then the acceleration $a = \mathbf{g}$ and initial velocity is U=0.

Substitute $S = H_{max}$, $U = 0$, $a = g$, $t = t_d$ in $S = Ut + \frac{1}{2}\mathbf{g}t^2$, we get

$H_{max} = 0(t) + \frac{1}{2}\mathbf{g}t_d^2$ -----> $\boxed{t_d = \sqrt{\frac{2H}{g}}}$

But body falling from maximum height. It is possible when only it is thrown up vertically up with initial velocity 'U'. So from $H_{max} = \frac{U^2}{2g}$ and $H_{max} = \frac{1}{2}\mathbf{g}t_d^2$

We get $\frac{U^2}{2g} = \frac{1}{2}\mathbf{g}t_d^2$

$\frac{U^2}{2g} = t_d^2$ ------> $\boxed{t_a = \frac{U}{g}}$

So for every vertically thrown up body Time of Ascent is equal to Time of Decent.

Time of flight:

It is the time for which a body thrown up vertically remains in air before it reaches ground.
So time of flight is the sum of time of ascent and time of flight.

Time of flight $T = t_a + t_d$ ---------> $T = \frac{U}{g} + \frac{U}{g}$ ------> $\mathbf{T = \frac{2U}{g}}$

----->Derive the expression for velocity of a body thrown up vertically on reaching point of projection:

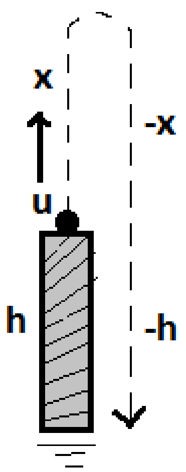
Let a body thrown up vertically, it can reach maximum height 'h'. from that maximum height it can falling freely.

Hence it has initial velocity U=0, acceleration $a = \mathbf{g}$.

Then final velocity of the body for reaching the ground is $V^2 - U^2 = 2aS$

$V^2 - 0^2 = 2\mathbf{g}h$ -----> $V = \sqrt{2gh}$

Derive an equation of motion for a body thrown up vertically from the top of a Tower:



Let a body thrown up vertically from the top of a tower of height 'h' with a initial velocity 'U'.

After some time 't' it can reach the ground, hence acceleration $a = -\mathbf{g}$

Total displacement $S = x - x - h = -h$

Therefore the equation of motion is $S = Ut + \frac{1}{2}\mathbf{g}t^2$ -----> $-h = Ut - \frac{1}{2}\mathbf{g}t^2$

$h = -Ut + \frac{1}{2}\mathbf{g}t^2$

and maximum height reached by the body from the ground (H) is $H = h + x$
H = height of the tower + Max height reached by the body above the top of the tower.
$H = h + \frac{U^2}{2g}$

Projectile:

A body thrown up at an angle other than 90^o to the horizontal (x-axis) under the action of gravity is called projectile. The angle at which the body is projected is known as angle of projection.

Ex: 1)cricket ball thrown by a fielder.
2)Javelin thrown by an athlete.
3)Bomb thrown from a moving aeroplane.

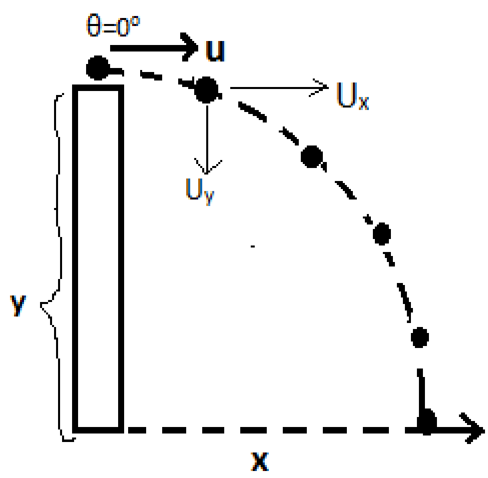
There are two types of projectile motions . 1) Horizontal projection.
2)Oblique projection.

A projectile which has 0^o angle of projection with the horizontal (x-axis) from certain height, then the projection is known as horizontal projection. A projectile which has angle of projection other than 90^o and 0^o with the horizontal, then it is called oblique projection. The path of the projectile is known as trajectory.

------->Show that the path of a horizontally projected body is a parabola?

Let a body thrown horizontally with a velocity 'U' from certain height 'y'. After 't' seconds it reaches ground at a distance 'x'. In the path of the projectile the body will have two velocity components.

In horizontal direction the body has uniform velocity and zero acceleration, because there is no any influence of gravity on acceleration of the body.



In vertical direction the body has non uniform velocity, because here the acceleration of the body is influenced by the gravity.

In horizontal direction, uniform velocity $U_x = U\cos\theta = U\cos 0^o = U$

Distance moved $S = X$

Acceleration $a = 0$, time = t

Therefore horizontal distance moved $S = U\,t + \frac{1}{2}a\,t^2$

$X = U\,t + 0$ ------> $t = \frac{X}{U}$

In vertical direction, velocity $U_y = U\sin\theta = U\sin 0^o = 0$

Distance moved $S = y$, Acceleration $a = \mathbf{g}$, time = t

Vertical distance moved $S = Ut + \frac{1}{2}a\,t^2$

$$Y = U_y\,t + \frac{1}{2}a\,t^2$$

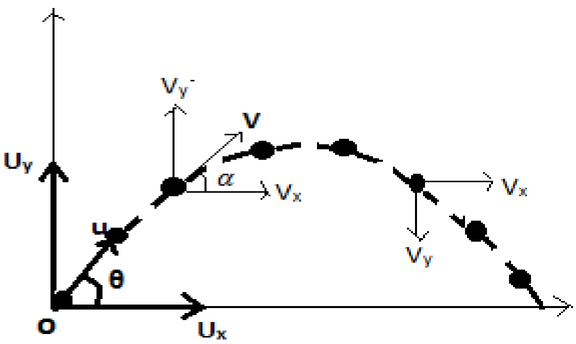
$$Y = 0\,(t) + \frac{1}{2}g\left(\frac{X}{U}\right)^2$$

$$\boxed{Y = \left(\frac{\mathbf{g}}{2U^2}\right)X^2}$$

Therefore $y = AX^2$, here $A = \frac{\mathbf{g}}{2U^2}$ is a constant.

The above equation represents parabola, hence the path of a horizontal projectile is a parabola.

--------->Show that the path of a oblique projected body is a parabola?



Let a body thrown up vertically at an angle 'θ' to the horizontal with initial velocity 'U'.

The point of projection is called as origin (o). In the path of the body it's velocity is resolved into two components.

They are horizontal component of velocity $\mathbf{U_x} = U\cos\theta$

Vertical component of velocity $\mathbf{U_y} = U\sin\theta$

So that velocity $\overline{U} = U_x\,\hat{\imath} + U_y\,\hat{\jmath} = U\cos\theta\,\hat{\imath} + U\sin\theta\,\hat{\jmath}$

In horizontal direction U x remains constant because there is no influence of gravity.

Here velocity $U_x = U\cos\theta$, time = t, acceleration a = 0, distance travelled s = x

So from $s = Ut + \frac{1}{2}a\,t^2 \rightarrow x = (U\cos\theta) + 0$

$$T = \frac{X}{U\cos\theta} \quad \text{(or)} \quad \frac{X}{U_x}$$

In vertical direction velocity u_y changes continuously, because there is an influence of gravity on acceleration, that is de acceleration. Here velocity $U_y = U\sin$, time = t, acceleration a = - g, distance travelled s = y.

So from $s = Ut + \frac{1}{2}a\,t^2$

$$Y = (U\sin\theta)t - \frac{1}{2}\mathbf{g}\,t^2$$

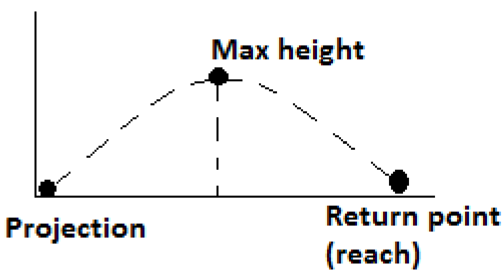
$$Y = (U\sin\theta)\left(\frac{X}{U\cos\theta}\right) - \frac{1}{2}\mathbf{g}\left(\frac{X}{U\cos\theta}\right)^2$$

$$Y = (\tan\theta)\,x - \left(\frac{g}{2U^2\cos^2\theta}\right)X^2 \text{ --------->} \; Y = A\,X - B\,X^2$$

The above equation represents the equation of parabola. Hence the path of a Oblique projectile is a parabola.

------>Explain the parameters of a Obliquely projected body? (or) Derive the expressions for maximum height, time of ascent, time of descent, time of flight, Range of a Obliquely projected body?

Maximum height reached by a projectile:



Let a body thrown up obliquely at an angle ' ' to the horizontal with an initial velocity 'U'.

At maximum height the final velocity becomes Zero and acceleration is a = -**g**.

So here $V_y = 0$, $U = U_y = U\sin\theta$, $a = -\mathbf{g}$, $S = H_{max}$

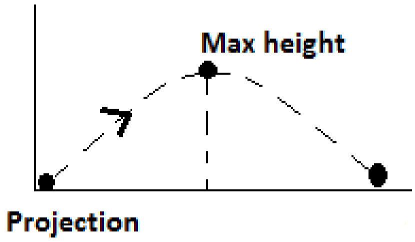
Using $V_y^{\,2} - U_y^{\,2} = 2aS$ ----> $0^2 - (U\sin\theta)^2 = -2g\,H_{max}$

$$\boxed{H_{max} = \frac{U^2\sin^2\theta}{2g}}$$

Time of ascent: " It is the time taken by a body to reach the maximum height. "

Let a body thrown up obliquely with an angle 'θ' to the horizontal with an initial velocity U.

At maximum height $V_y = 0$



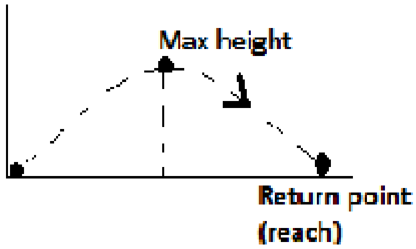
Vertical component of initial velocity $U_y = U \sin\theta$

Acceleration $a = -\mathbf{g}$, time $t = t_a$

Using $V = U + at$, we get $V_y = U_y + (-\mathbf{g}\, t_a)$

$0 = U \sin\theta - \mathbf{g}\, t_a$ -----> $\boxed{t_a = \frac{U \sin\theta}{\mathbf{g}}}$

Time of Descent:



"It is the time taken by a body to reach the ground from maximum height."

At maximum eight initial velocity $U_y = 0$.

Acceleration $a = g$, time $t = t_d$, distance $S = H$ max

Using $S = Ut + \frac{1}{2} a t^2$, we get $H_{max} = U_y t_d + \frac{1}{2}\mathbf{g}\, t_d^2$

$H_{max} = (0)\, t_d + \frac{1}{2}\mathbf{g}\, t_d^2$

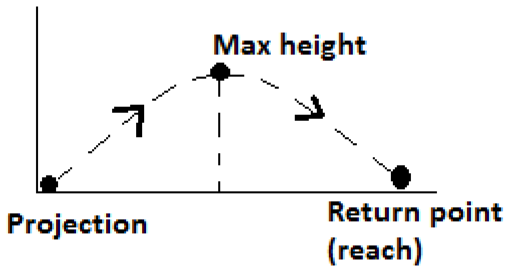
$\frac{U^2 \sin^2\theta}{2\mathbf{g}} = \frac{1}{2}\mathbf{g}\, t_d^2$---------> $\boxed{t_d = \frac{U \sin\theta}{\mathbf{g}}}$

Time of flight:

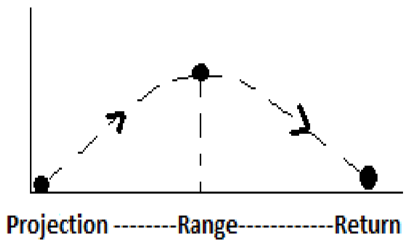
Time taken by a projectile to return to the same plane of it's projection is known as the time of flight.

So time of flight $T = t_a + t_d = \frac{U \sin\theta}{g} + \frac{U \sin\theta}{g}$

$\boxed{T = \frac{2U \sin\theta}{g}}$



Horizontal distance travelled (or) Range of a projectile:



It is a distance travelled by s body from point of projection to point of return on a same plane.

Let a body projected obliquely with an angle ' θ ' in a plane. After 'T' seconds the body return at some point on a same plane.

Horizontal range = Horizontal component of velocity X time of flight.

$R = U_x\, T$

$R = U \cos\theta \times \frac{2U\sin\theta}{g} = \frac{U^2(2\sin\theta\cos\theta)}{g}$ ----> $\boxed{R = \frac{U^2 \sin 2\theta}{\mathbf{g}}}$

For the angles 30° and 60° the body can have same range.

--------->Show that for maximum range angle of projection is 45°

Horizontal distance travelled by a body (or) Range of a projectile is R = $\frac{U^2 \sin 2\theta}{g}$

It is maximum when only sin2 is maximum that is sin2θ = 1

$2\theta = \sin^{-1}(1)$

$2\theta = \pi/2$ -------> $\theta = \frac{\pi}{4} = 45°$

So for to travel maximum range the angle of projection is 45°.

PROBLEMS:1)A stone is allowed to fall freely from the top of tower 300m high and at the same time another stone is projected vertically upwards with a velocity of 75m/sec. when and where stones will meet.

(Hint: use $S = Ut + \frac{1}{2}at^2$ for two times **Ans**: t=4sec, h = 221.6m)

2)A ball is thrown at an angle 30° to the horizontal with an initial velocity of 20 m/sec. Find its maximum height and Horizontal Range. -------2 times **Ans:** $H_{max} = 5.1\ m$, R = 35.35 m

3)A football is projected at an angle 30° to the horizontal with an initial velocity of 29.4 m/sec. Find its maximum height . **Ans:** $H_{max} = 11.025\ m$

4)An aeroplane flying horizontally with a speed of 360 km/hr releases a bomb at a height of 490m from the ground. Find when and where the bomb will strike the ground. **Ans:** x =1000m, t= 10sec

5)An aeroplane flying horizontally with a speed of 180 km/hr releases a bomb at a height of 490m from the ground. Find when and where the bomb will strike the ground. **Ans:** x =500m, t= 10sec

6)A stone is thrown vertically upwards from the ground with a velocity of 14 m/sec. Find the maximum height and time of flight. ---------2 times **Ans:** $H_{max} = 10\ m$, T = 2.86sec

7)When a body is projected, if the maximum height reached and horizontal range are equal, what would be the angle of projection. **Ans:** $\theta = \tan^{-1}(4)$ = 75.96°

8)A stone is projected vertically upwards from the top of a tower with velocity 9.8m/sec. It reached the ground after 6sec. What is the height of the tower. **Ans:** h = 117.6m

9) A stone is projected vertically upwards from the top of a tower with velocity 25m/sec. It reached the ground after 20 sec. What is the height of the tower. **Ans:** h =1460 m