ECET – ECE – 2024 – solutions

51) $\rightarrow$ $L \propto V^x A^y F^z$

Take dimensions on bothsides

$$[M^1L^2T^{-1}] = [M^0L^1T^{-1}]^x \times [L^1T^{-2}]^y \times [M^1L^1T^{-2}]^z$$

$$[M^1L^2T^{-1}] = [M^z L^{x+y+z} T^{-x-2y-2z}]$$ [4]

Compare powers on bothsides, we get

$z=1$, $x+y+z=2$, $-x-2y-2z=-1$

$x+y=1$ $\quad$ $-x-2y=1$

$-y=2 \Rightarrow y=-2$

$x+(-2)=1 \Rightarrow x=3$

$\therefore$ Angular momentum $L \propto V^3A^{-2}F^1$ [3]

52) $\rightarrow$ $V = At^2 + Bt + C$ $\longrightarrow$ From Homogeneity

$At^2 =$ velocity

$A = \frac{\text{velocity}}{t^2} = \frac{m/sec}{sec^2} = \frac{m}{sec^3}$

53) $\rightarrow$ $\bar{C} = \bar{A} + \bar{B}$, $|\bar{C}| = |\bar{A}| + |\bar{B}|$ $\longrightarrow$ (or) $C = A+B$ [3]

$|\bar{C}| = |\bar{A}+\bar{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$

$C^2 = A^2+B^2+2AB\cos\theta$

$(A+B)^2 = A^2+B^2+2AB\cos\theta$

$\cancel{A^2}+\cancel{B^2}+\cancel{2AB} = \cancel{A^2}+\cancel{B^2}+\cancel{2AB}\cos\theta$

$1 = \cos\theta \longrightarrow \theta = 0^\circ$ [3]

54) Triangle Area $= \frac{|\bar{A}\times\bar{B}|}{2} = \frac{1}{2}\left|\begin{matrix}\hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & 4 & 0\end{matrix}\right|$

$= \frac{1}{2}|(8-3)\hat{k}|$

$= \frac{5}{2}|\hat{k}| = \frac{5}{2}(1) = 2.5$

55) $\rightarrow$ $\underset{v_1}{\square} \xrightarrow[a]{t_1} \underset{2v_1}{\square} \xrightarrow[a]{t_2} \underset{3v_1}{\square}$ [1]

$v = u + at \longrightarrow 2v_1 = v_1 + at_1$, $3v_1 = v_1 + at_2$

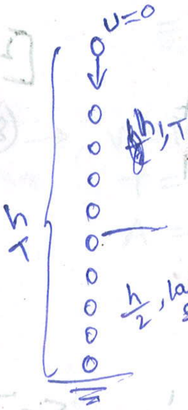
$v_1 = at_1$ $\qquad$ $2v_1 = at_2$

$2(at_1) = at_2$

$t_2 = 2t_1$

(2)

56) → If a body travels half of its total path in the last second of its fall from rest then the height of its fall is ____ ($g = 9.8\,m s^{-2}$) [1]



$S = ut + \frac{1}{2}at^2$

$h = 0(T) + \frac{1}{2}gT^2$

$h = \frac{1}{2}gT^2 \rightarrow (1)$

$h_1 = 0 + \frac{1}{2}g(T-1)^2 \rightarrow (2)$

distance travelled in last second is

$\frac{h}{2} = h - h_1 \rightarrow h_1 = \frac{h}{2}$

$\frac{g(T-1)^2}{2} = \frac{gT^2}{4}$

$T^2 - 4T + 2 = 0$

$\Rightarrow T = 2 + \sqrt{2}$

$\therefore$ height, $h = \frac{1}{2} \times 9.8 \times (2+\sqrt{2})^2 = 57.18\,m$

57) → $\theta = 45^\circ$ [2]

$H = 30$

$R = ?$

$H = \frac{u^2 \sin^2\theta}{2g} = 30 \rightarrow \frac{u^2 \sin^2 45^\circ}{2g} = 30$

$\rightarrow \frac{u^2}{4g} = 30$

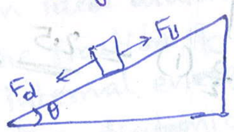
$\rightarrow \frac{u^2}{g} = 120.$

$\therefore R = \frac{u^2 \sin 2\theta}{g} = 120 \times \sin 2(45^\circ) = 120\,m$

58) → friction is independent of Area [1]

$\therefore \mu = 0.2$ unchange.

59) → The force required just to move a body up an inclined plane is double the force required just to prevent the body sliding down it. If the coefficient of friction is $\frac{1}{\sqrt{3}}$, then the angle of the plane is ____. [4]



$F_u = 2F_d$, $\mu = \frac{1}{\sqrt{3}}$, $\theta = ?$

$mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$

$\sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta$

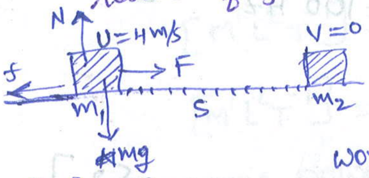
$\sin\theta = 3\mu\cos\theta$

$\tan\theta = 3\mu$

$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\theta = 60^\circ$

60) → If an ice block of mass 42 kg moves with initial velocity 4 m/s on a rough surface of coefficient of friction 0.1, then the amount of ice melted as a result of friction before the block comes to rest is [2]



workdone (W) on a ice block creates heat in it.

Heat will melt the ice → $Q = m_2 L$ → (1)

L = Latentent heat $= 334000$ J/kg.

$v^2 - u^2 = 2as$

$0 - u^2 = -2\mu g s$,

$s = \frac{u^2}{2\mu g} = \frac{16}{2 \times 0.1 \times 9.8}$

$s = 8$ m

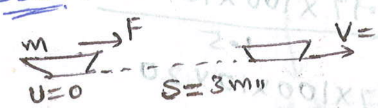
Work $W = F \times S = f \times S = \mu N S = \mu m_1 g S$ → (2)

work = Heat → $\mu m_1 g s = m_2 L$

$\Rightarrow m_2 = \frac{\mu m_1 g s}{L} = \frac{0.1 \times 42 \times 9.8 \times 8}{334000} = 0.001$ kg

$m_2 = 1$ gm.

61) → [2]



$V^2 - U^2 = 2aS$

$V = \sqrt{2aS}$

$F = ma \rightarrow a = \frac{F}{m} = \frac{5 \times 10^4}{3 \times 10^7} = \frac{5}{3} \times 10^{-3}$

$\therefore V = \sqrt{2 \times \frac{5}{3} \times 10^{-3} \times 3} = \sqrt{10^{-2}} = 0.1$ m/s.

62) → $W = \bar{F} \cdot \bar{S} = (2,4,5) \cdot (3,2,1)$ [4]

$= 6 + 8 + 5$

$W = 19$ Joule.

63) → Power $P = 45$ HP $= 45 \times 746$ watt. [1]

velocity $v = 15$ m/s

Force $F = ?$

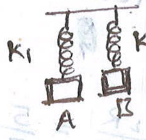
$P = \frac{W}{t} = \frac{F \cdot S}{t} = F \cdot v$

$F = \frac{P}{V} = \frac{45 \times 746}{15}$

$= 3 \times 746$

$F = 2238$ N

64) → spring constants $K_1 = m\omega_1^2$, $K_2 = m\omega_2^2$, equal masses [3]



velocities are equal $v = r_1\omega_1 = r_2\omega_2$

amplitude ratios $\frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$

$\frac{r_1}{r_2} = \frac{\sqrt{K_2/m}}{\sqrt{K_1/m}} = \sqrt{\frac{K_2}{K_1}}$

65) $T = 1$ sec. [1]

$g = 10$ ms^{-2}

$l = ?$

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \frac{gT^2}{4\pi^2}$

$= \frac{10 \times (1)^2}{4 \times (3.14)^2}$

$= 0.2535$ m

66) $\rightarrow T = 2\pi\sqrt{\frac{l}{g}} \rightarrow$ free fall $g=0 \rightarrow T=\infty$ [4]

67) $\rightarrow n' = \left(\frac{v}{v-v_s}\right) n$ [1]

$= \left(\frac{v}{v-\frac{v}{10}}\right) \times 90 = \frac{10}{9} \times 90 = 100$ Hz.

68) $\rightarrow V = 100 \times 10 \times 30\ m^3$ [2]

$T = 1.5$ sec

$A = ?$ $\quad T = \frac{0.17\,V}{A}$

$A = \frac{0.17 \times 100 \times 10 \times 30}{1.5}$

$= 0.17 \times 100 \times 10 \times 20$

$A = \underline{3400}$

70) $\rightarrow m = 14$ gm [2]

$M =$ nitrogen $(N_2) = 28$

$\therefore$ gas equation $PV = nRT$

$n = \frac{m}{M} = \frac{14}{28} = \frac{1}{2} \rightarrow PV = \frac{1}{2}RT$

71) $P_1 = 80$ [3]

$m_1 = 1$

$m_2 = \frac{3}{5}$

$P_2 = ?$

initial gas  $m=1$

Now  $\frac{3}{5}$ Remains. $\frac{2}{5}$ leaks

$V =$ constant
$T =$ constant
$M =$ same gas (constant)
$R = 8.31$ constant.

$PV = \frac{m}{M}RT \Rightarrow P \propto m$

$\Rightarrow \frac{P_1}{m_1} = \frac{P_2}{m_2} \Rightarrow \frac{80}{1} = \frac{P_2}{3/5}$

$\Rightarrow P_2 = 80 \times \frac{3}{5} = 16 \times 3 = 48$ cm of Hg.

72) An ideal diatomic gas is heated at constant pressure. The fraction of the heat energy supplied to increase the internal energy of the gas is [4]

Sol) For diatomic gas $\quad dU = dQ + dW$

$C_v = \frac{5}{2}R$

$C_p = \frac{7}{2}R$ $\quad$ Ask, $\frac{dU}{dQ} = \frac{n c_v dT}{n c_p dT} = \frac{C_v}{C_p} = \frac{5/2\,R}{7/2\,R} = \frac{5}{7}$

73) The distance between the atoms of a diatomic gas remains constant. Then its molecular specific heat at constant volume is [1]

$C_v = \frac{f}{2}R$ $\quad f \rightarrow$ degrees of freedom

$f = 3$ translational $+ 2$ Rotational

$f = 5$

$\therefore C_v = \frac{5}{2}R$