ECET Study Material (1)

(physics – 25 Marks)

Lesson (1) – Units & Dimensions :-

UNITS:

	MKS	CGS	FPS
mass	Kg	gm	pound
Legth	metre	cm	Foot
Time	Sec	Sec	Sec

SI UNITS Dimensions — Dimension

mass → Kg → M

Length → metre → L

Time → second → T

Temperature → °C, °F → K (kelvin)

current → Amphere → A

Luminous Intensity → candela → cd

Amount of substance → mole

Supplementary Units { Plane angle → Radians; Solid angles → steradian }

Unit conversions

mass: 1 Kg = 1000 gm ⇒ 1 gm = $\frac{1\,Kg}{1000}$

1 Km = 1000 ms →

1 m = 100 cm →

1 cm = 10 mm →

1 hour = 60 minute = 60×60 = 3600 sec.

1 minute = 60 sec.

velocity = $\frac{\text{distance}}{\text{time}} \rightarrow \frac{m}{sec}$

acceleration = $\frac{\text{velocity}}{\text{time}} \rightarrow \frac{m}{sec^2}$

Force = mass × acceleration

$F = ma \rightarrow Kg \times \frac{m}{sec^2} \rightarrow$ Newton (MKS, SI)

$\rightarrow gm \times \frac{cm}{sec^2} \rightarrow$ dyne (CGS)

work (energy) = Force × displacement

$W = F \times S = Kg \times \frac{m^2}{sec^2} \rightarrow$ Joule (MKS)

$= gm \times \frac{cm^2}{sec^2} \rightarrow$ erg (CGS)

∴ 1 Newton = 1000 gm × $\frac{100\,cm}{sec^2}$

1 Newton = 10^5 dyne ⇒ 1 dyne = $\frac{1\,Newton}{10^5}$

like that 1 Joule = 10^7 erg ⇒ 1 erg = $\frac{1\,Joule}{10^7}$

(T) Tera → Trillion → 10^{12} times.

(G) Giga → Billion → 10^9 times — 10^7 → crore

(M) Mega → million → 10^6 times — 10^5 → lakh

(K) Kilo → 1000 times → 10^3

~~micro → $\frac{1}{1000}$ = 0.001 times → 10^{-3}~~

(m) milli → $\frac{1}{1000}$ = 0.001 times → 10^{-3}

(μ) micro → 10^{-6} times

(n) nano → 10^{-9}

(p) Pico → 10^{-12}

(f) Fermi → 10^{-15}

Energy Conversion

Chemical – 1 Calorie = 4.2 Joule → Mechanical

Electrical – 1 KWH = 36×10^5 Joule → Mech

Electrical – 1 eV = 1.6×10^{-19} Joule → Mech

practice BITS:

1) Young's modulus of steel is $2 \times 10^{11}\,N/m^2$. Its value in dyne/cm² is ——

Sol $2 \times 10^{11} \frac{N}{m^2} = \frac{2 \times 10^{11} \times 10^5\,dyne}{(100\,cm)^2} = \frac{2 \times 10^{16}\,dyne}{10^4\,cm^2} = 2 \times 10^{12}\,dyne/cm^2$.

2) The value of planck's constant 'h' is 6.626×10^{-34} Joule × Hz^{-1}. Its value in eV (electron volt) is ——.

Sol $h = 6.626 \times 10^{-34} \frac{Joule}{sec}$, 1 eV = 1.6×10^{-19} Joule. ⇒ 1 Joule = $\frac{1\,eV}{1.6 \times 10^{-19}}$

$\therefore h = 6.626 \times 10^{-34} \times \frac{1\,eV}{1.6 \times 10^{-19}} = 4.1 \times 10^{-15}\,eV$.

→ Derivation of a New physical equation

Ex: 1) If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be ——.

mass $m \propto V^x F^y E^z$

$m = \text{constant} \times V^x F^y E^z$

take dimensions bothside & compare powers.

$\Rightarrow [M^1L^0T^0] = 1[M^0L^1T^{-1}]^x \times [M^1L^1T^{-2}]^y \times [M^1L^2T^{-2}]^z$

$[M^1L^0T^0] = [M^{0+y+z}\,L^{x+y+2z}\,T^{-x-2y-2z}]$

compare the individual powers.

$1 = 0+y+z$, $x+y+2z = 0$, $-x-2y-2z = 0$

$y+z = 1$

$-2+(y+z)+z = 0$

$-2+1+z = 0$

$z = 1$

$y = 0$

$-x-2(1) = 0$

$x = -2$

∴ mass → $m \propto V^{-2} F^0 E^1$

1. UNITS & Dimensions

Date: .... Page No.: ....
Experiment No.: ....

2020-ECE, Mech → The unit of angular frequency is radians/sec (or) Degrees/sec.

2020-CIVIL → Young's Modulus of Steel is $2\times10^{11}\,N/m^2$. Its value in dyne/cm² is —

$$2\times10^{11}\,N/m^2 \rightarrow \frac{2\times10^{11}\times10^{5}\,dyne}{(100\,cm)^2} \rightarrow \frac{2\times10^{16}\,dyne}{10^{4}\,cm^2} \rightarrow 2\times10^{12}\,dyne/cm^2$$

2020 EEE → Dimension of velocity gradient is $[M^0L^0T^{-1}]$

$$\hookrightarrow \frac{dv}{dx} \rightarrow \frac{m/sec}{m} \rightarrow \frac{1}{sec} \rightarrow M^0L^0T^{-1}$$

2020 Mech → The dimensional formula for magnetic flux is ______

✓(a) $M^1L^2T^{-2}A^{-1}$ (b) $M^1L^3T^{-2}A^{-2}$ (c) $M^1L^{-2}T^{-2}A^{-2}$ (d) $M^1L^2T^{-1}A^2$

2021-CIVIL → The dimension of universal gas constant 'R' is ______

ideal gas eq. $PV = nRT \rightarrow R = \frac{PV}{nT} \rightarrow \frac{\text{Pressure}\times\text{volume}}{\text{mole}\times\text{temperature}}$

dimension $R \rightarrow \frac{[M^1L^{-1}T^{-2}]\cdot[L^3]}{n\cdot[K]} \rightarrow [M^1L^2T^{-2}K^{-1}]$

2021-ECE → The dimensional formula for gravitational Constant, G is ______

Newton's force $F = \frac{Gm_1m_2}{R^2} \rightarrow G = \frac{FR^2}{m_1m_2} \rightarrow \frac{[M^1L^1T^{-2}]\times[M^0L^2T^0]}{[M^1L^0T^0][M^1L^0T^0]} \rightarrow [M^{-1}L^3T^{-2}]$

2021-EEE → The dimension of universal gas constant 'R' is : $[M^1L^2T^{-2}K^{-1}]$

2021-Mech → The dimensional formula for gravitational constant, G is $[M^{-1}L^3T^{-2}]$

2022-CIVIL → The dimensions of permeability is ______

electromagnetic force $F = \frac{\mu_0}{4\pi}\frac{m_1m_2}{r^2} \rightarrow \mu_0 = \frac{4\pi Fr^2}{m_1m_2} \rightarrow \frac{\text{Newton}\times m^2}{(\text{Amp}\times m)^2}$

$$\rightarrow \frac{[M^1L^1T^{-2}][M^0L^2T^0]}{[M^0L^1T^0A^1]^2} \rightarrow [M^1L^1T^{-2}A^{-2}]$$

2022-ECE → parsec is the unit of Distance → length

2022-EEE → Dimensions of permeability is $[M^1L^1T^{-2}A^{-2}]$

2022-Mech → parsec is the unit of Distance.

2021-CIVIL, EEE → The value of planck's constant 'h' is $6.626\times10^{-34}\,J\cdot Hz^{-1}$. Its value in eV is ______

$h = 6.626\times10^{-34}\,J\cdot Hz^{-1}$ (or) Joule.sec , $1eV = 1.6\times10^{-19}$ Joule.

$$\therefore h = \frac{6.626\times10^{-34}}{1.6\times10^{-19}}\,eV \rightarrow h = 4.14\times10^{-15}\,eV.$$

2021-ECE, EEE, Mech → which of the following quantities have not been expressed in proper units?

(a) electric field → Newton/coulomb (b) Surface Tension = Newton/meter

✓(c) energy → kg m/sec (d) pressure = Newton/m²

option (c) is not correct

Hoco?

2022-CIVIL, EEE

If velocity (v), force (F) and energy (E) are taken as fundamental units, then dimensional formula for <u>mass</u> will be ______.

$$m \propto v^x F^y E^z \Rightarrow [M^1L^0T^0] = [M^0L^1T^{-1}]^x \times [M^1L^1T^{-2}]^y \times [M^1L^2T^{-2}]^z$$

$$[M^1L^0T^0] = [M^0L^xT^{-x}] \times [M^yL^yT^{-2y}] \times [M^zL^{2z}T^{-2z}]$$

powers $\rightarrow 1 = y+z,\quad 0 = x+y+2z,\quad 0 = -x-2y-2z$

$$x+y+2z=0,\quad \cancel{x+2y}$$
$$-x-2y-2z=0$$
$$-y=0 \rightarrow y=0 \Rightarrow 1=y+z \rightarrow 1=0+z \rightarrow z=1$$

$\therefore 0 = x+y+2z \rightarrow 0 = x+0+2(1) \rightarrow x=-2$

$\therefore$ eq. of mass $m \propto v^{-2}F^0E^1$.

2022-ECE, Mech

Among the following pairs, which pair <u>does not</u> have identical dimensions. } check all.

✓ (a) moment of inertia & moment of force (b) Work and Torque.

(c) angular momentum & planck constant (d) Impulse & Momentum.

(a) moment of inertia $I = mr^2 \rightarrow kg \times m^2$ } Unequal ✓

Moment of force $C = F\times d \rightarrow kg\times m^2/sec^2$

(b) Work $W = F\times S \rightarrow kg\times m^2/sec^2$ } same.

Torque $\tau = F\times r \rightarrow kg\times m^2/sec^2$

(c) angular Momentum $L = P\times r = mvr \rightarrow kg\times m^2/sec$ } same.

Planck constant $h = E/\gamma \rightarrow \frac{kg\times m^2/sec^2}{1/sec} \rightarrow kg\times\frac{m^2}{sec}$.

(d) Impulse $i = F\times t \rightarrow kg\times\frac{m}{sec^2}\times sec \rightarrow kg\times\frac{m}{sec}$ } same.

Momentum $P = m\times v \rightarrow kg\times\frac{m}{sec}$

2023: The dimension of the ratio of angular momentum and linear momentum is –

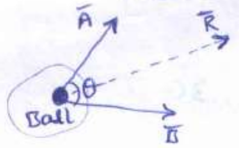
$L = mvr$, $P = mv$; $\frac{L}{P} = r \rightarrow$ metre $\rightarrow M^0L^1T^0$

2023: one Fermi is equivalent to ___10^{-15}___ meter.

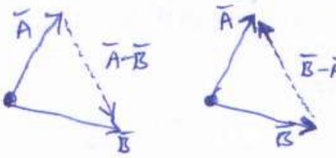
2. VECTORS

Vector → magnitude (value) changes with direction (+, −, θ, axis, plane).
3D vector → $\bar{A} = 3\hat{i}+4\hat{j}+6\hat{k} \to x\hat{i}+y\hat{j}+z\hat{k}$ → volume (space) indicates.
2D vector → $\bar{A} = 3\hat{i}+4\hat{j}$ → xy plane.
In general $\bar{A} = |\bar{A}|\hat{n}$ → magnitude & direction.
here $\hat{n} = \frac{\bar{A}}{|\bar{A}|} = \frac{\text{vector}}{\text{magnitude}}$ → Unit vector (direction purpose).
Vectors → velocity, acceleration, force, momentum, Torque, weight...
Scalars → time, distance, mass, speed, work, energy, power, ...
Parallelogram, Triangle law of vectors → gives resultant vectors of given 2 vectors.
↳ add, subtraction.

Parallelogram law:



here addition of $\bar{A}, \bar{B}$ is at centre
$\bar{R} = \bar{A}+\bar{B}$ → vector form
Resultant magnitude
$|\bar{R}| = |\bar{A}+\bar{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$
direction $\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$

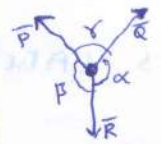


Subtraction at edges
$|\bar{R}| = |\bar{A}-\bar{B}| = |\bar{B}-\bar{A}| = \sqrt{A^2+B^2-2AB\cos\theta}$

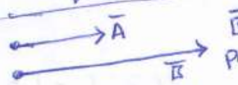
Triangle law

$\bar{AB}+\bar{BC} = \bar{AC}$
$\bar{P}+\bar{Q} = \bar{R}$
$\bar{P}$ continues with $\bar{Q}$ at B

Lami Theorem (Ratios)



$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$
equillibrium of the body.



$\bar{B} = 2\bar{A}$ parallel $\theta = 0°$

perpendicular → $\theta = 90°$

Antiparallel (opposite) $\theta = 180°$

PRACTICE BITS

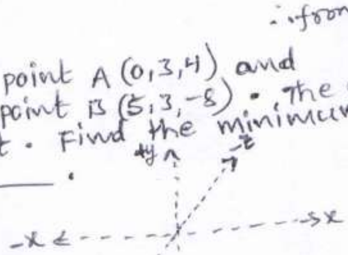
1) Unit vector of $\bar{A} = 2\hat{i}-\hat{j}+2\hat{k}$ is ——.
$\hat{n} = \frac{\bar{A}}{|\bar{A}|} = \frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{2^2+1^2+2^2}} = \frac{2\hat{i}-\hat{j}+2\hat{k}}{3} = \frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}+\frac{2}{3}\hat{k}$.

2) If two vectors $\bar{A}$ and $\bar{B}$ such that $|\bar{A}-\bar{B}| = |\bar{A}+\bar{B}|$ then $\bar{A}$ is —— to $\bar{B}$
$|\bar{A}-\bar{B}| = |\bar{A}+\bar{B}| \Rightarrow \sqrt{A^2+B^2-2AB\cos\theta} = \sqrt{A^2+B^2+2AB\cos\theta} \Rightarrow 4AB\cos\theta = 0$
$4\neq0, A\neq0, B\neq0$ so $\cos\theta = 0$ $\therefore \theta = 90°$.
∴ $\bar{A}$ is perpendicular to $\bar{B}$.

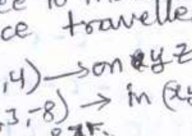
that is for any two perpendicular vectors, their addition (total) & subtraction values are equal.
that is the diagonals are equal → square, rectangle.

3) The resultant of two forces 3P and 2P is R. If the first force is doubled, then the resultant is also doubled. The angle between the two forces is ——.
$R = \sqrt{(3P)^2+(2P)^2+12P^2\cos\theta} \to ①$
$R^2 = 13P^2+12P^2\cos\theta$
$12P^2\cos\theta = R^2-13P^2$
Now $2R = \sqrt{(6P)^2+(2P)^2+24P^2\cos\theta}$
$4R^2 = 40P^2+24P^2\cos\theta \to ②$
$2R^2 = 20P^2+12P^2\cos\theta$
$2R^2 = 20P^2+R^2-13P^2 \Rightarrow R^2 = 7P^2$
∴ from (1) $12P^2\cos\theta = 7P^2-13P^2$
$12P^2\cos\theta = -6P^2 \Rightarrow \cos\theta = -\frac{1}{2}$
$\theta = 120°$

4) A cat is situated at point A (0,3,4) and a rat is situated at point B (5,3,−8). The cat is free to move but is rat is always at rest. Find the minimum distance travelled by cat to catch is rat is ——.

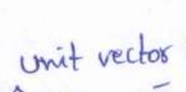


A(0,3,4) → on yz plane → 2D
B(5,3,−8) → in (xy-z) space → 3D



CAT Moves towards RAT that is $|\bar{A}-\bar{B}|$, edge
$|\bar{A}-\bar{B}| = \sqrt{(0-5)^2+(3-3)^2+(4+8)^2} = \sqrt{25+0+144} = \sqrt{169} = 13$ metr.

5) Find unit vector perpendicular to $\bar{A} = \hat{i}+\hat{j}-\hat{k}$, $\bar{B} = 2\hat{i}-\hat{j}+3\hat{k}$ is ——.



$\hat{n}$ → is perpendicular to both $\bar{A}, \bar{B}$.
$\hat{n}$ → is perpendicular vector of $\bar{A}$ & $\bar{B}$.
$\bar{A}\times\bar{B}$ gives the perpendicular vector = Magnitude × direction.
↳ $\bar{C}$ that is $\bar{C} = |\bar{C}|\hat{n}$
$\hat{n} = \frac{\bar{C}}{|\bar{C}|} = \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|}$

$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 2\hat{i}-5\hat{j}-3\hat{k} \to \hat{n} = \frac{2\hat{i}-5\hat{j}-3\hat{k}}{\sqrt{4+25+9}} = \frac{2\hat{i}-5\hat{j}-3\hat{k}}{\sqrt{38}}$.

2·VECTORS

Experiment No. : Date : Page No. :

2020 ECE, Mech

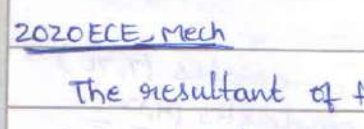
The sum of two vectors A and B is at right angles to their difference, then
(a) A = B (b) A = 2B (c) B = 2A (d) A & B have same direction.

Sum A + B is passing as diagonal (middle)
difference A − B is passing from end points of A, B

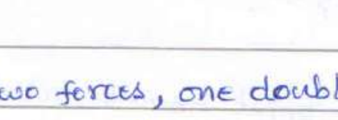
in parallelogram these two are diagonals. and angle between these two is $\neq 90^\circ$
But question says A+B, A−B have right angle, that is
it is possible in square only → All sides equal → A = B.

2020 ECE, Mech

The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces, The angle between two forces is ____



(or)



(a) 120° (b) 60° (c) 90° (d) 150° Ans: 120°

$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)$

$\alpha = 90^\circ,\ Q = 2P,\ \theta = \theta$

$\tan 90^\circ = \frac{2P\sin\theta}{P+2P\cos\theta}$

$\frac{1}{0} = \frac{2P\sin\theta}{P(1+2\cos\theta)}$

$1+2\cos\theta = 0$

$\cos\theta = -\frac{1}{2},\ \theta = 120^\circ$

(or)

$\alpha = 90^\circ,\ Q = 2P,\ \theta \to 90+\theta$

$\tan 90^\circ = \frac{2P\sin(90+\theta)}{P+2P\cos(90+\theta)}$

$\frac{1}{0} = \frac{2P\cos\theta}{P-2P\sin\theta} \Rightarrow P - 2P\sin\theta = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

$\therefore 90+\theta \to 90+30 \to 120^\circ$.

2020 CIVIL, EEE

Unit vector parallel to the resultant of vectors $\bar{A} = 4\hat{i} - 3\hat{j}$ and $\bar{B} = 8\hat{i} + 8\hat{j}$ will be ____.

resultant vector is $\bar{R} = \bar{A} + \bar{B} \to \bar{R} = 12\hat{i} + 5\hat{j}$

Unit vector $\hat{n} = \frac{\bar{R}}{|\bar{R}|} = \frac{12\hat{i}+5\hat{j}}{\sqrt{144+25}} = \frac{12\hat{i}+5\hat{j}}{13}$.

2020 CIVIL, EEE

The resultant of two forces 3P and 2P is R. If the 1st force is doubled, then the resultant is also doubled. The angle between two force is ____.

$R, 3P, 2P \to R = \sqrt{(3P)^2+(2P)^2+2(3P)(2P)\cos\theta} \Rightarrow R = \sqrt{13P^2+12P^2\cos\theta}$

$2R, 6P, 2P \to 2R = \sqrt{(6P)^2+(2P)^2+2(6P)(2P)\cos\theta} \Rightarrow 2R = \sqrt{40P^2+24P^2\cos\theta}$

Squaring $\to$ $R^2 = 13P^2+12P^2\cos\theta$, $4R^2 = 40P^2+24P^2\cos\theta$ $\Big\} \to 4(13P^2+12P^2\cos\theta) = 40P^2+24P^2\cos\theta$

$52P^2+48P^2\cos\theta = 40P^2+24P^2\cos\theta$

$12P^2+24P^2\cos\theta = 0 \Rightarrow 1+2\cos\theta = 0 \Rightarrow \cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$

2021 CIVIL, EEE

A unit vector perpendicular to $\bar{A} = \hat{i}+\hat{j}-\hat{k}$ and $\bar{B} = 2\hat{i}-\hat{j}+3\hat{k}$ is ____

$\bar{A}\cdot\bar{C} = 0 \to (\hat{i}+\hat{j}-\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k}) = 0 \to x+y-z = 0$

$\bar{B}\cdot\bar{C} = 0 \to (2\hat{i}-\hat{j}+3\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k}) = 0 \to 2x-y+3z = 0$

$\Rightarrow \frac{x}{\begin{vmatrix}1 & -1\\ -1 & 3\end{vmatrix}} = \frac{y}{\begin{vmatrix}-1 & 1\\ 3 & 2\end{vmatrix}} = \frac{z}{\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}}$, $x = 2,\ y = -5,\ z = -3$

$\therefore \bar{C} = 2\hat{i}-5\hat{j}-3\hat{k}$, $\hat{n} = \frac{\bar{C}}{|\bar{C}|} = \frac{2\hat{i}-5\hat{j}-3\hat{k}}{\sqrt{4+25+9}} = \frac{2\hat{i}-5\hat{j}-3\hat{k}}{\sqrt{38}}$

2021-CIVIL, EEE

If the two vectors $\bar{A}$ and $\bar{B}$ are such that $|\bar{A}-\bar{B}| = |\bar{A}+\bar{B}|$ then

(a) $\bar{A}=\bar{B}$ (b) $\bar{A}$ is parallel to $\bar{B}$ (c) $|\bar{B}|=0$ ✓(d) $\bar{A}$ is perpendicular to $\bar{B}$

$|\bar{A}-\bar{B}| = |\bar{A}+\bar{B}| \longrightarrow \sqrt{A^2+B^2-2AB\cos\theta} = \sqrt{A^2+B^2+2AB\cos\theta}$

$\longrightarrow 4AB\cos\theta = 0 \longrightarrow \theta = 90^\circ \longrightarrow \bar{A} \perp \bar{B}$

2021-ECE, Mech

A vector $\bar{A}$ is along positive x-axis. If $\bar{B}$ is another vector such that $\bar{A}\times\bar{B}$ is Zero, then $\bar{B}$ would be ___

(a) $4\hat{j}$ ✓(b) $-4\hat{i}$ (c) $-(\hat{i}+\hat{j})$ (d) $\hat{j}+\hat{k}$.

$\bar{A}$ is x-axis, $\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|\sin\theta = 0 \longrightarrow \theta = 0^\circ$ (or) 180°

if $\theta = 0^\circ \longrightarrow \bar{B} = 4\hat{i}$ satisfies $\longrightarrow$ not given

$\theta = 180^\circ \longrightarrow \bar{B} = -4\hat{i}$ satisfies $\longrightarrow$ given

2021-ECE, Mech

The scalar product of two vectors is $2\sqrt{3}$ and vector product is 2 then angle between them is ___.

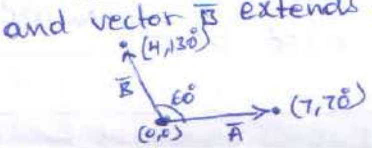
$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta = 2\sqrt{3}$

$\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|\sin\theta = 2$

$\longrightarrow \frac{\cos\theta}{\sin\theta} = \frac{2\sqrt{3}}{2} \longrightarrow \cot\theta = \sqrt{3} \longrightarrow \theta = \cancel{60^\circ}\ \underline{30^\circ}$

2022-CIVIL, EEE

Vector $\bar{A}$ extends from the origin to a point having polar coordinates $(7, 70^\circ)$ and vector $\bar{B}$ extends from the origin to a point having polar coordinates $(4, 130^\circ)$. Find $\bar{A}\cdot\bar{B}$.



$(r,\theta) \longrightarrow (7,70^\circ)\ (4,130^\circ)$

$|\bar{A}| = 7 \quad |\bar{B}| = 4, \quad \theta = 130-70 = 60^\circ$

$\therefore \bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta = 7\times4\times\cos60^\circ = 28\times\frac{1}{2} = \underline{14}$

2022-CIVIL, EEE

If two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $-4\hat{i}-6\hat{j}-\lambda\hat{k}$ are parallel to each other then value of λ be ___

(a) 2 (b) 4 (c) 0 (d) 6

$\theta = 0^\circ \longrightarrow \bar{A}\times\bar{B} = \bar{0}$

$\begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ -4 & -6 & -\lambda \end{vmatrix} = 0 \Rightarrow \hat{i}(-3\lambda-6) - \hat{j}(-2\lambda-4) + \hat{k}(-12+12) = 0\hat{i}+0\hat{j}+0\hat{k}$

$-3\lambda-6=0 \quad -2\lambda-4=0$

$\lambda = \underline{\underline{-2}}. \quad \lambda = \underline{\underline{-2}}.$ Answer not given

(or)

$\theta = 0^\circ \longrightarrow \bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos 0^\circ$

$-8-18+\lambda = \sqrt{4+9+1}\cdot\sqrt{16+36+\lambda^2}$

$(\lambda-26)^2 = 14(52+\lambda^2) \Rightarrow \lambda^2-52\lambda+26^2 = 14\times52+14\lambda^2$

$\Rightarrow 13\lambda^2+52\lambda+52 = 0 \longrightarrow \lambda^2+4\lambda+4=0 \longrightarrow (\lambda+2)^2=0 \longrightarrow \lambda = \underline{\underline{-2}}.$

(or)

parallel means $\bar{A} = x\bar{B}$ or $\bar{B} = x\bar{A}$

$\bar{A} \longrightarrow 2,3,-1 \longrightarrow -2(2,3,-1) \to -4,-6,2$

$\bar{B} \longrightarrow -4,-6,-\lambda \longrightarrow -4,-6,-\lambda$

$\therefore -\lambda = 2 \to \lambda = \underline{\underline{-2}}.$

2022-ECE, Mech

one of the two forces is double the other and their resultant is equal to the greater force. The angle between them is ___.

given $P,\ Q=2P,\ R=Q$

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$Q = \sqrt{P^2+4P^2+4P^2\cos\theta}$

$(2P)^2 = P^2+4P^2+4P^2\cos\theta$

$4P^2 = 5P^2+4P^2\cos\theta$

$-P^2 = 4P^2\cos\theta \longrightarrow \cos\theta = -\frac{1}{4}$

$\longrightarrow \theta = \underline{\cos^{-1}\left(\frac{-1}{4}\right)}.$

2022-ECE, Mech

If three vectors $\bar{A} = \hat{i}-2\hat{j}+3\hat{k}$, $\bar{B} = x\hat{i}+3\hat{k}$, $\bar{C} = 7\hat{i}+3\hat{j}-11\hat{k}$ are coplanar, then $x =$ ___

$\hookrightarrow [\bar{A}\ \bar{B}\ \bar{C}] = 0 \Rightarrow \det = 0 \Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ x & 0 & 3 \\ 7 & 3 & -11 \end{vmatrix} = 0 \Rightarrow 1(-9)+2(-11x-21)+3(3x) = 0$

$\Rightarrow -9-22x-42+9x = 0$

$\Rightarrow -9-13x-42 = 0 \Rightarrow 13x = -51$

$x = -51/13$

DOT (scalar) product

$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta = \text{scalar}$
vector vector

ex:- work $W = \bar{F}\cdot\bar{S} = |\bar{F}||\bar{S}|\cos\theta$
power $P = \bar{F}\cdot\bar{V}$

$\hat{i}\cdot\hat{i} = 1$, $\hat{j}\cdot\hat{j} = 1$, $\hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = 0$, $\hat{j}\cdot\hat{k} = 0$, $\hat{k}\cdot\hat{i} = 0$

$\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z$ = scalar.

$\theta = 90^\circ \rightarrow \bar{A}\cdot\bar{B} = 0 \rightarrow$ min
$\theta = 0^\circ \rightarrow \bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\times 1 \rightarrow$ max

CROSS (vector product)

$\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|\sin\theta$ = vector. $\rightarrow \bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \rightarrow$ determinant value.

ex:- Torque $\bar{\tau} = \bar{F}\times\bar{r}$ $\rightarrow$ Area of parallelogram $= |\bar{A}\times\bar{B}|$
linear velocity $\bar{V} = \bar{r}\times\bar{\omega}$, Area of Triangle $= \frac{|\bar{A}\times\bar{B}|}{2}$.
Angular Momentum $L = \bar{P}\times\bar{r} = mvr$

$\hat{i}\times\hat{i} = 0$, $\hat{j}\times\hat{j} = 0$, $\hat{k}\times\hat{k} = 0$, $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{i} = -\hat{k}$ .....

PRACTICE BITS

1) vector $\bar{A}$ extends from the origin to a point having polar coordinates $(7, 70^\circ)$ and vector $\bar{B}$ extends from the origin to a point having polar coordinates $(4, 130^\circ)$. Find $\bar{A}\cdot\bar{B}$ = ______.

$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta$
$= 7\times 4\times\cos 60^\circ = 28\times\frac{1}{2} = \underline{14}$.

2) Find the values of x & y for which vectors $\bar{A} = 6\hat{i} + x\hat{j} - 2\hat{k}$ and $\bar{B} = 5\hat{i} - 6\hat{j} + y\hat{k}$ may be parallel.

Method (1)
parallel $\rightarrow$ same ratios.

$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$

$\frac{6}{5} = \frac{x}{-6} = \frac{-2}{y}$

$x = -36/5$, $\frac{6}{5} = \frac{-2}{y} \Rightarrow y = \frac{-5}{3}$.

Method (2)
$|\bar{A}\times\bar{B}| = |\bar{A}||\bar{B}|\sin 0^\circ$ = vector
$|\bar{A}\times\bar{B}| = \bar{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

$\begin{vmatrix} i & j & k \\ 6 & x & -2 \\ 5 & -6 & +y \end{vmatrix} = \bar{0} \Rightarrow (xy-12)\hat{i} - (6y+10)\hat{j} + (-36-5x)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

$\therefore xy - 12 = 0$, $6y + 10 = 0$, $-36 - 5x = 0$
$y = \frac{-5}{3}$, $x = -\frac{36}{5}$.

3) Find $\hat{i}\times(\hat{j}\times\hat{k}) = ?$
$\hat{i}\times(\hat{j}\times\hat{k}) = \hat{i}\times\hat{i} = 0$

(4) Find $\hat{j}\cdot(\hat{k}\times\hat{i})$
$\hat{j}\cdot(\hat{k}\times\hat{i}) = \hat{j}\cdot(-\hat{j}) = -(\hat{j}\cdot\hat{j}) = -1$

Lesson-3 - Dynamics

equations of motion

1) $V = U + at$
2) $S = Ut + \frac{1}{2}at^2$
3) $V^2 - U^2 = 2aS$
4) $S_n = U + a(n - \frac{1}{2})$

average velocity $v = \frac{x}{t}$ & $v = \frac{v_1 + v_2}{2}$

t = time
U = initial velocity
V = final velocity
a = acceleration
S = distance after t sec
S_n = distance in n^{th} sec.

Freely falling body
$U = 0$, $a = +g$
1) $V = gt$
2) $S = \frac{1}{2}gt^2$
3) $V^2 = 2gS$
4) $S_n = g(n - \frac{1}{2})$

vertically thrown up
$V = 0$, $a = -g$
1) $0 = U - gt$
2) $S = Ut - \frac{1}{2}gt^2$
3) $0 - U^2 = -2gS$
4) $S_n = U - g(n - \frac{1}{2})$.

PRACTICE BITS

1) A particle is projected vertically upward with a speed of 40 m/s, then the velocity of the particle 2 seconds before it reaches the maximum height is ______ here ($g = 10\ m/s^2$)

$U = 40\ m/s$, time of Ascent $\rightarrow$ time to take max height.
$t = \frac{U}{g} = \frac{40}{10} = 4$ sec.
So $4 - 2 = 2$ sec to take that much height.
$\therefore$ velocity after 2 secs (lower part) $\rightarrow V = U - gt$
$= 40 - 10\times 2$
$V = 40 - 20 = 20$ m/sec

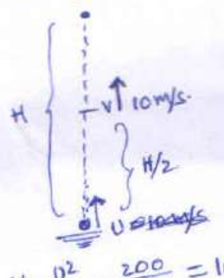
(or) from upper part. $V = 0$, 2 sec, U: $V = U - gt$
$0 = U - 10\times 2 \Rightarrow U = 20$ m/sec.

$V = 0$, 2 sec, $V = ?$, 2 sec, 4 sec, $U = 40$ m/s.

2) A body is thrown vertically upwards with a velocity. Select the incorrect statements from the following.
(a) Both velocity & acceleration are zero at its highest point.
(b) velocity is max & acceleration is zero at the highest point.
(c) velocity is max and acceleration is 'g' downwards at its heighest point.

(1) a, b and c ✓
(2) b and c
(3) a and b.
(4) a and c.

(3) A stone is thrown vertically upwards, when stone is at half of its max height, its speed is 10 m/s, then the max height attained by the stone is ____ $(g = 10\ m/s^2)$

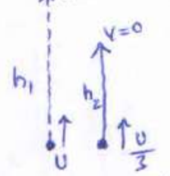


use $v^2 - u^2 = 2aS$ two times.

for total Height $v = 0,\ a = -g,\ S = H$
$0 - U^2 = -2gH \Rightarrow H = \frac{U^2}{2g}$

for half of its max height
$v = 10,\ a = -g,\ S = H/2$
$10^2 - U^2 = -2g \cdot \frac{H}{2}$
$100 - U^2 = -10H$
$100 - U^2 = -10 \times \frac{U^2}{2g} \to 10$
$100 - U^2 = -\frac{U^2}{2}$
$100 = U^2 - \frac{U^2}{2} \Rightarrow \frac{U^2}{2} = 100 \Rightarrow U^2 = 200 \Rightarrow \therefore H = \frac{U^2}{2g} = \frac{200}{2 \times 10} = 10$ metre.

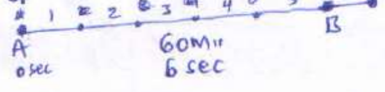
(4) A stone thrown vertically upwards with a speed of 'U' m/sec attains a height h_1. Another stone thrown vertically upwards from the same point with a speed of $\frac{U}{3}$ m/sec attains a height h_2. Then $h_2 =$ ____.



$v^2 - U^2 = 2aS$
$0 - U^2 = -2gh_1$
$h_1 = \frac{U^2}{2g}$

$v^2 - U^2 = 2aS \Rightarrow 0 - \left(\frac{U}{3}\right)^2 = -2gh_2$
$\frac{U^2}{9} = 2gh_2 \Rightarrow h_2 = \frac{U^2}{9 \times 2g} \Rightarrow \underline{h_2 = \frac{h_1}{9}}$.

(5) A car moving with constant acceleration covered the distance between two points 60 m apart in 6 sec. Its speed as it passes the second point was 15 m/s. The acceleration is ____.



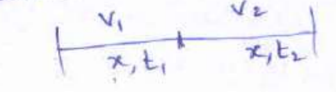
average velocity $v = \frac{v_1 + v_6}{2}$

$\frac{\text{Total distance}}{\text{Total time}} = \frac{v_1 + 15}{2}$

$\frac{60}{6} = \frac{v_1 + 15}{2} \Rightarrow v_1 + 15 = 20 \Rightarrow v_1 = 5$ m/sec.

$\therefore$ acceleration $a = \frac{dv}{dt} = \frac{v_6 - v_1}{t_2 - t_1} \Rightarrow a = \frac{15-5}{6-0} = \frac{10}{6} = \frac{5}{3}\ m/s^2$

(6) A person ~~tt~~ travelling on a straight line moves with a uniform velocity v_1 for a distance 'x' and with a uniform velocity v_2 for the next equal distance. The average velocity v is ____.



here $v_1 = \frac{x}{t_1}$, $v_2 = \frac{x}{t_2}$ now total time $t = t_1 + t_2$

ave velocity $v = \frac{x+x}{t} \Rightarrow v = \frac{2x}{t}$

$\therefore t = t_1 + t_2 \Rightarrow \frac{2x}{v} = \frac{x}{v_1} + \frac{x}{v_2} \Rightarrow \frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2} \Rightarrow \underline{v = \frac{2v_1v_2}{v_1+v_2}}$.

(7) The velocity of a body moving along a straight line with uniform deacceleration 'a' reduces by $\frac{3}{4}$ of its initial velocity. The total time of motion of the body is ____

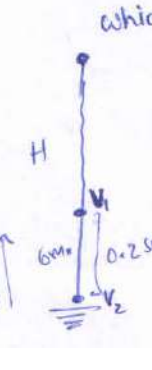
$a = \frac{dv}{dt} \Rightarrow a = \frac{V-U}{t_2 - t_1}$, dt always +ve.
deacceleration $\Rightarrow -a$

velocity reduces by $\frac{3}{4}$ means $\to V = \frac{U}{4}$ present velocity.

$\therefore -a = \frac{\frac{U}{4} - U}{t} \Rightarrow t = \frac{-3U/4}{-a} \Rightarrow \underline{t = \frac{3U}{4a}}$.

(8) A ball is dropped from a height H. If it takes 0.2 sec to cross the last 6 m before hitting the ground, the value of height H from which it was dropped is ____.

$H_{max} = \frac{U^2}{2g}$, we know, thrown up velocity = hitting velocity on ground (free fall)
$U = V_2 \Rightarrow H_{max} = \frac{V_2^2}{2g}$



average velocity $v = \frac{x}{t} = \frac{v_1 + v_2}{2}$

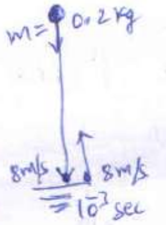
$\frac{6}{0.2} = \frac{v_1 + v_2}{2} \Rightarrow v_1 + v_2 = 60$

and $v = u + at$
$v_2 = v_1 + gt \Rightarrow v_2 - v_1 = 10 \times 0.2 = 2$

$2v_2 = 62 \Rightarrow v_2 = 31$

$\therefore H_{max} = \frac{(31)^2}{2 \times 10} = \frac{31 \times 31}{20} = \frac{961}{20} = \underline{48.05\ m}$.

9) A rubber ball of mass 0.2 kg falls onto the floor. The ball hits with a speed of 8 m/sec and rebounds with exactly the same speed, high speed photographs show that the ball is in contact with the floor for 10^{-3} sec. Then the average force exerted on the ball by the floor is ——.



$$F = ma = m\left(\frac{v-u}{t}\right)$$
$$= 0.2 \times \left(\frac{8-(-8)}{10^{-3}}\right)$$
$$= 0.2 \times \frac{16}{10^{-3}} = 200 \times 16 = 3200\ N.$$

PROJECTILE → 2D motion → curve → 2 components.

Oblique projectile: velocity can be resolved (x, y) as $U\cos\theta$, $U\sin\theta$

After few seconds it can have another velocity 'v' with angle α it is also resolved as $v\cos\alpha$, $v\sin\alpha$

final velocity $v = \sqrt{U^2 + g^2t^2 - 2Ugt\cos\theta}$

$v\cos\alpha = U\cos\theta$

$v\sin\alpha = U\sin\theta - gt$

height $H_{max} = U^2\sin^2\theta / 2g$

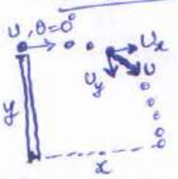
time of Ascent (Descent) $t = \frac{U\sin\theta}{g}$

Time of Flight $T = \frac{2U\sin\theta}{g}$

Horizontal distance (Range) $R = \frac{U^2\sin 2\theta}{g}$

the path is parabola.

Horizontal projectile:



$U_x = U\cos\theta$, $U_y = U\sin\theta$

time to reach ground is $t = \frac{x}{U\cos\theta}$, the path is parabola $\Rightarrow y = Ax^2 \Rightarrow y = \left(\frac{g}{2U^2\cos^2\theta}\right) \cdot x^2$.

PRACTICE BITS

1) A projectile is fired with a speed 'U' at an angle 'θ' with the horizontal. Find its speed when its direction of motion makes an angle α with the horizontal. ?

$U_x = U\cos\theta$, $U_y = U\sin\theta$

$v_x = v\cos\alpha$, $v_y = v\sin\alpha$

We know $v = u + at$ → In Horizontal direction, there is no change in velocity ⇒ a = 0

∴ $v_x = U_x + 0 \Rightarrow v\cos\alpha = U\cos\theta$.

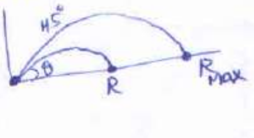
$v = \frac{U\cos\theta}{\cos\alpha}$

$v = U\cos\theta\sec\alpha$.

In vertical direction, velocity changes due to gravity (g).

∴ $v_y = U_y - gt \Rightarrow v\sin\alpha = U\sin\theta - gt$.

2) A bullet fired from a gun falls at a distance half of its Maximum Range. The angle of projection of the bullet is ——.



$R = \frac{U^2\sin 2\theta}{g}$ for Max Range → means $\sin 2\theta = 1 \Rightarrow \theta = \frac{90^\circ}{2} = 45^\circ$

∴ $R_{max} = \frac{U^2}{g}$, Now if it projected with an angle 'θ' and $R = \frac{R_{max}}{2}$

$$\frac{U^2\sin 2\theta}{g} = \frac{U^2}{2g}$$
$$\sin 2\theta = 1/2$$
$$2\theta = 30^\circ \longrightarrow \theta = 15^\circ.$$

3) During a projection motion, if the Max height is equal to the horizontal range, then the angle of projection with the horizontal is ——.

$$H_{max} = R \Rightarrow \frac{U^2\sin^2\theta}{2g} = \frac{U^2\sin 2\theta}{g} \Rightarrow \frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$
$$\Rightarrow \sin\theta = 4\cos\theta$$
$$\Rightarrow \tan\theta = 4$$
$$\Rightarrow \theta = \tan^{-1}(4).$$

8

4) The horizontal range of a projectile is $4\sqrt{3}$ times of its Max height. Its angle of projection will be ___.

$$R = 4\sqrt{3}\, H_{max} \Rightarrow \frac{U^2 \sin 2\theta}{g} = 4\sqrt{3} \cdot \frac{U^2 \sin^2\theta}{2g}$$

$$2\sin\theta\cos\theta = 2\sqrt{3}\sin^2\theta$$

$$\cancel{2}\cos\theta = \sqrt{3}\sin\theta \Rightarrow \cot\theta = \sqrt{3} \Rightarrow \theta = 30^\circ.$$

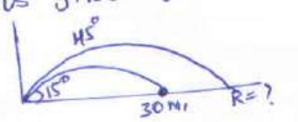
5) A body is projected at an angle otherthan 90° with the horizontal with some velocity. If the t_a of the body is 1 sec, then the Max height it can reach is ___ $(g = 10\, m/s^2)$

$t_a = \frac{U\sin\theta}{g} = 1$

$U\sin\theta = g = 10$ m/sec.

$$H_{max} = \frac{U^2\sin^2\theta}{2g} = \frac{(10)^2}{2\times 10} = \frac{100}{20} = 5 \text{ m}$$

6) The range of a projectile fired at an angle of 15° is 30 m. If it is fixed with the same speed at an angle of 45°, its range will be ___

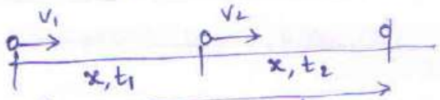


$$R_1 = \frac{U^2 \sin 2\theta}{g} \Rightarrow 30 = \frac{U^2\sin 2(15^\circ)}{g} \Rightarrow 30 = \frac{U^2}{2g} \Rightarrow \frac{U^2}{g} = 60$$

$$\text{Now } R_2 = \frac{U^2 \sin 2(45^\circ)}{g} = \frac{U^2(1)}{g} \Rightarrow R_2 = \frac{U^2}{g} = 60 \text{ meter (double)}.$$

7) A person travelling on a straight line moves with a uniform velocity v_1 for a distance 'x' and with a uniform velocity v_2 for the next equal distance. The average velocity (v) is given by ___

(a) $v = \frac{v_1 + v_2}{2}$ (b) $v = \sqrt{v_1 v_2}$ ✓(c) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ (d) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$



$v_1 = \frac{x}{t_1}$, $v_2 = \frac{x}{t_2}$, $v = \frac{2x}{t_1+t_2}$ (or) $v = \frac{2x}{t}$ (or) $v = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$

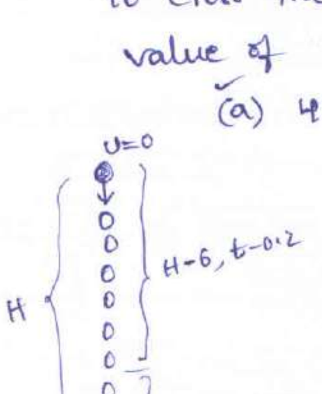
Total time $t = t_1 + t_2$

$$\frac{2x}{v} = \frac{x}{v_1} + \frac{x}{v_2}$$

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}.$$

8) A ball is dropped from a height H. If it takes 0.2 sec to cross the last 6.0 m before hitting the ground, the value of height H from which it was dropped.

✓(a) 48 m (b) 42 m (c) 12 m (d) 30 m



use $s = ut + \frac{1}{2}at^2 \rightarrow$ 2 times.

$H = (0)t + \frac{1}{2}gt^2 \rightarrow H = \frac{1}{2}gt^2$

& $H - 6 = (0)(t - 0.2) + \frac{1}{2}g(t-0.2)^2$

$H - 6 = \frac{g}{2}\left(t^2 - \frac{4t}{10} + \frac{4}{100}\right)$

$\cancel{H} - 6 = \cancel{\frac{1}{2}gt^2} - \frac{gt}{5} + \frac{g}{50}$

$-6 = \frac{-gt}{5} + \frac{0.98}{5}$

$-30 = -gt + 0.98$

$gt = 30 + 0.98$

$t = \frac{30.98}{g} = \frac{30.98}{9.8}$

$t = 3.16$ sec.

$\therefore H = \frac{1}{2}gt^2$

$H = \frac{9.8}{2} \times (3.16)^2$

$= 4.9 \times 3.16 \times 3.16$

$H = 48$ m.

3-Dynamics

Experiment No. : Date : Page No. :

2020-ECE, Mech

A body starts from rest travels a distance 'x' in 1st two seconds and a distance 'y' in next two seconds. The relation between x and y is ___

(a) $y = 4x$ (b) $y = x$ (c) $y = 3x$ (d) $y = 2x$

$U=0$, $t=2$, x; $t=4$ sec, y

$S = Ut + \frac{1}{2}at^2$

$t=2$, $U=0$, $S=x$ $\Rightarrow x = 0 + \frac{1}{2}a(2)^2 \rightarrow x = 2a$ after 2 secs.

$t=4$, $U=0$, $S=y$ $\Rightarrow y = \frac{1}{2}a(4)^2 = 8a$ after 4 secs.

in next 2 secs $\Rightarrow y = 8a - 2a$, $y = 6a$.

$\therefore x = 2a,\ y = 6a \longrightarrow y = 3x$.

2020-ECE, Mech

Two bodies are projected from the ground with the same speed. If the angles of their projection from the ground are 45° and 15°. The ratio of ranges is ___

$R = \frac{U^2 \sin 2\theta}{g} \rightarrow \frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2} \rightarrow \frac{R_1}{R_2} = \frac{\sin 90^\circ}{\sin 30^\circ} \rightarrow \frac{R_1}{R_2} = \frac{1}{1/2} \rightarrow R_1 : R_2 = 2:1$

2020-ECE, Mech

Two bodies of different masses are dropped from heights of 2m and 8m. The ratio of the time taken by them is ___

m_2, $U=0$, 8m; m_1, $U=0$, 2m

$S = Ut + \frac{1}{2}at^2$

$2 = 0(t) + \frac{1}{2}gt_1^2,\ 8 = 0(t) + \frac{1}{2}gt_2^2 \Rightarrow \frac{2}{8} = \frac{\frac{g}{2}t_1^2}{\frac{g}{2}t_2^2} \Rightarrow \frac{1}{4} = \left(\frac{t_1}{t_2}\right)^2$

$\Rightarrow \frac{t_1}{t_2} = \frac{1}{2} \Rightarrow 1:2$

2020-ECE, CIVIL, EEE, Mech

The angle of projection of a projectile for which the horizontal range and maximum height are equal is ___

$H_{max} = R \rightarrow \frac{U^2 \sin^2\theta}{2g} = \frac{U^2 \sin 2\theta}{g} \Rightarrow \frac{\sin^2\theta}{2} = \sin 2\theta$

$\Rightarrow \frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta \Rightarrow \frac{\sin\theta}{2} = 2\cos\theta$

$\Rightarrow \tan\theta = 4 \rightarrow \theta = \tan^{-1}(4)$

2020-CIVIL, EEE

A particle is projected vertically upward with a speed of 40 m/s, then the velocity of the particle 2 sec before it reaches the maximum height is ___ (Take $g = 10\,m/s^2$)

v_4, v_3, v_2, v_1, $g = -10$, $U = 40$

time of Ascent $t_a = \frac{U}{g} = \frac{40}{10} = 4$ sec

to reach max height it will take 4 sec time.

$\therefore$ velocity at 2 sec time $V = U - gt$

$= 40 - 10(2)$

$= 40 - 20$

$V = 20$ m/sec

2020 - CIVIL, EEE

A car moving with constant acceleration covered the distance between two points 60 m apart in 6 sec. Its speed as it passes the 2nd point was 15 m/sec. The acceleration is —

(a) $\frac{1}{3}ms^{-2}$ (b) $\frac{2}{3}ms^{-2}$ (c) $\frac{3}{5}ms^{-2}$ ✓(d) $\frac{5}{3}ms^{-2}$.

V_1 [C] → 60 m, 6 sec $V_2 = 15$ m/sec.

$a = \frac{dv}{dt} = \frac{V_2 - V_1}{t}$, $S = ut + \frac{1}{2}at^2$

$a = \frac{15 - V_1}{6}$, $60 = 6V_1 + \frac{1}{2}a(6)^2$

$60 = 6V_1 + 18a$

$10 = V_1 + 3a$

$V_1 = 10 - 3a$

$a = \frac{15-(10-3a)}{6}$

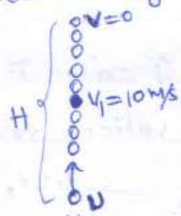
$6a = 15 - 10 + 3a$

$6a = 5 + 3a \Rightarrow 3a = 5$

$\Rightarrow a = 5/3$.

2020 - CIVIL, EEE

A stone is thrown vertically upwards, When stone is at half of its maximum height, its speed is $10\,ms^{-1}$. then the maximum height attained by the stone is —— ($g = 10\,ms^{-2}$)



$V^2 - U^2 = 2aS$

$0 - U^2 = -2gH$

$H = \frac{U^2}{2g}$

now. $V = 0$, $U = V_1$, $a = -g$, $H = \frac{H}{2}$

$V^2 - U^2 = 2aS$

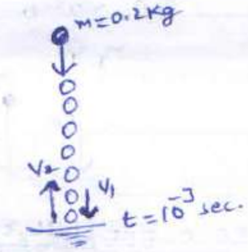
$0 - V_1^2 = -2g \times \frac{H}{2} \Rightarrow V_1^2 = gH \Rightarrow H = \frac{V_1^2}{g}$

$\Rightarrow H = \frac{(10)^2}{10} \rightarrow H = 10$ meters

2021 - CIVIL

A rubber ball of mass 0.2 kg falls onto the floor. The ball hits with a speed of 8 m/s and rebounds with approximately the same speed. High speed photographs show that the ball is in contact with the floor for 10^{-3} sec. Then the average force exerted on the ball by the floor is ——.

(a) 1600 N (b) 160 N ✓(c) 3200 N (d) 320 N.



force exerted by the floor is –

$F = \frac{dp}{dt} = \frac{mV_2 - mV_1}{t_2 - t_1}$

but $V_2 = -V_1$

$F = \frac{-mV_1 - mV_1}{10^{-3}} = \frac{-2mV_1}{10^{-3}}$

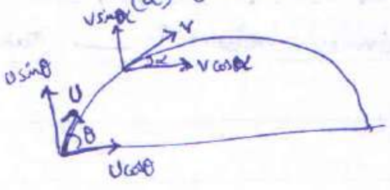
$= -2 \times 0.2 \times 8 \times 1000$

$F = -3200$ Newton.

2021 - CIVIL

A projectile is fired with a speed 'U' at an angle θ with the horizontal. Find its speed when its direction of motion makes an angle 'α' with the horizontal.

(a) $U\cos\theta\cos\alpha$ (b) $U\cos\theta$ (c) $U\cos\alpha$ ✓(d) $U\cos\theta\sec\alpha$.



No gravity along x-axis → $V\cos\alpha = U\cos\theta$

$\rightarrow V = \frac{U\cos\theta}{\cos\alpha}$

$V = U\cos\theta\sec\alpha$.

along y-axis there is gravity.

$\therefore V = U - gt$

$V\sin\alpha = U\sin\theta - gt$

Signature of the Co-Examiner

Name, Designation

Y. Veerendranath, M.Sc

Name, Designation & Address

ECET- FRICTION - PHYSICS

→ It is the contact (or) surface force between two bodies.
→ A force which opposes the relative motion between two surfaces in contact.
→ A force which oppose the motion of one body over the surface of another body.
→ UNITS → Newton.
→ Ex:- between Road & vehicles tyres, Tightning thread,
→ cause of friction is → interlocking of projections on both surfaces.
→ Rough surface means → high friction → motion decrease.
Smooth " " → low " → " increases.

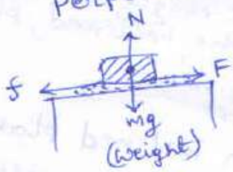
→ f 

→ Static friction → f_s → Max friction, body not moving.
Limiting friction → body starts motion.
→ Kinetic friction → f_K → friction in motion (low) → body moving.
→ Rolling friction → f_r → for Rolling bodies → very low.
$f_s > f_K > f_r$ (or) $f_r < f_K < f_s$

→ friction is independent of Area of contact.
→ friction depends on nature of surfaces & lubricants used.
→ frictional force is directly proportional to Normal force.
$f \propto N \Rightarrow f = \mu N$ μ → coefficient of friction.
μ → inbetween 0 & 1
$\mu > 0,\ \mu < 1$

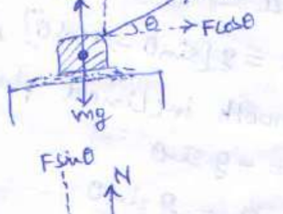
take leather, wood → $\mu_K = 0.125,\ \mu_s = 0.5$
Tyre, road → $\mu_K = 0.7,\ \mu_s = 1$ (high)
steel on steel → $\mu_K = 0.2,\ \mu_s = 0.6$
(Train tracks & wheels).
$\mu_s > \mu_K > \mu_r$

→ Normal force is the Reaction force pointing perpendicular to the contact surface.

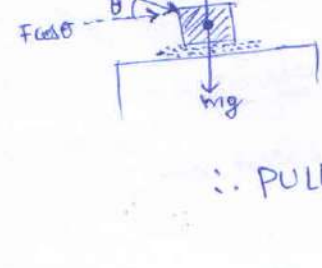


$N = mg$
$f = \mu N$
$f = \mu mg$

$N = mg\cos\theta < mg$
$\therefore N\downarrow$ hence $f\downarrow$
so that motion ↑
(vehicle on Bridges).



Newton's 3rd Law.
$N + F\sin\theta = mg$
$N = mg - F\sin\theta$
N↓ hence f↓ in pulling a body.
⇒ Motion increases.
Ex:- pulling Lan Roller, cart, vehicle engines in front side.



$N = mg + F\sin\theta$
N↑ hence f↑ in pushing a body.
⇒ Motion decreases
ex:- pushing door, sofa, Bus

∴ PULL a body is easy than PUSH a body.

2020-CIVIL, EE

A car ...
two points 6...
was 15 m/sec ...
(a) $\frac{1}{3}\,ms^{-2}$

V_1 [C] → 60m

2020-CIVIL, EEE

A stone ...
maximum ...
attained b...

$V=0$, $V_1=10$, H, U

2021-CIVIL

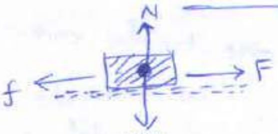
A rubbe...
speed of 8 m...
High speed ph...
Then the a...
(a) 16

2021-CIVIL

A proj...
horizontal ...
an angle ...
(a) U

$U\sin\theta$, $U\cos\theta$, $V\sin\alpha$, V

→ Motion of a body (vehicle) on Rough Horizontal Surface (Road) :-



force by engine $F = ma$

opposing frictional force $f = \mu N = \mu mg$

Newton 3rd Law $F = -f$

$ma = -\mu mg$

$a = -\mu g$

acceleration in magnitud $a = \mu g$

∴ equations of motion: final velocity $V = U - (\mu g)t$

$V^2 - U^2 = -2\mu g s$

$S = Ut - \frac{1}{2}\mu g t^2$

$S_n = U - \mu g(n - \frac{1}{2})$

if a body starts moving with initial velocity 'U', and after some time it comes to rest ($V=0$).

then $V = U - \mu g t$

$0 = U - \mu g t \longrightarrow \boxed{t = \frac{U}{\mu g}}$ → time of travel.

and distance travelled is $V^2 - U^2 = -2\mu g S$

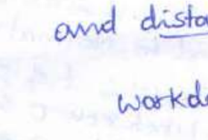
$0 - U^2 = -2\mu g S \Rightarrow \boxed{S = \frac{U^2}{2\mu g}}$

Workdone by the frictional force is $W = f \times S$

$W = \mu m g S$.

→ Angle of friction (on Rough Horizontal Surface)

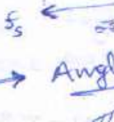
It is the angle made by resultant force and normal reaction.



$\tan\theta = \frac{f_s}{N} \Rightarrow \tan\theta = \mu_s \Rightarrow \theta = \tan^{-1}(\mu_s)$

$\theta\uparrow\ \mu_s\uparrow\ f_s\uparrow$.

→ Angle of Repose: (on Inclined plane)



It is the minimum angle of inclination to which the body starts sliding on inclined plane. (OR)

It refers to the max angle at which an object can rest (stay) on the inclined plane

$\theta_r = \tan^{-1}(\mu_s)$

→ If the body moving up on Rough inclined plane, acceleration of a body $a = -g(\sin\theta + \mu\cos\theta)$, increases

→ moving down, acceleration $a = g[\sin\theta - \mu\cos\theta]$, decreases.

→ If the body moving up on smooth inclined plane, acceleration of a body $a = -g\sin\theta$

→ moving down, acceleration $a = g\sin\theta$.

→ Friction uses:
1) used to walk & Run on road.
2) used to hold the objects in hand.
3) Matchstick lighted
4) working of ~~B~~ Machine belt, Chains.
5) Brake system.
6) nail fixing in walls & bricks.

→ Loses: 1) Due to friction, the parts of a body will damage in Accidents.
2) parts of a machine corrosion, then It's power, energy, efficiency can be decreases.

→ Minimizing methods: the surface → Rough turned into Smooth.
1) by polishing
2) Lubricants used like oils, grease.
3) Rolling bodies used → wheels, ball bearings, tyres . . .
4) streamlining → front surface of vehicles are in suitable shape to reduce air friction.

Types of Forces

Force → PUSH (or) PULL which changes the state of rest (or) motion

1) Gravitational force: — force between two massive bodies.

$F = G\frac{m_1 m_2}{r^2}$ → long range forces (upto ∞)

$G = 6.66\times10^{-11}\ N\ m^2/kg^2$.

acceleration due to gravity of a planet $g = \frac{GM}{R^2} = 9.8\ m/s^2$

if masses are doubled, distance halfed then force becomes —

$m_1 \to 2m_1$
$m_2 \to 2m_2$
$r \to r/2$

$\Rightarrow force = G\frac{2m_1 \times 2m_2}{(r/2)^2}$

$= 16\,G\frac{m_1 m_2}{r^2}$ → 16 times increase.

2) Electromagnetic force:

Force between two charges (or) poles.

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$, $F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$

$\epsilon_0 = 8.85\times10^{-12}$, $\mu_0 = 4\pi\times10^{-7}$

Ex:- → electrons bounded in a Atom due to protons. → 230 Newtons.

→ Frictional force between two contact surfaces is due to EM force between Atoms.

→ Tension in a String is due to EM force at the ends.

→ Forces in a String.

(3) Strong force (or) Nuclear force:

Forces inside the nucleus to bind protons & neutro[ns] to hold the nucleus stable.

These are responsible for radioactivity, nuclear fission, nuclear fusion.

4) Weak Nuclear force

forces between protons, neutrons, electrons.

β^- decay ⇒ neutron changes itself into proton, electron & antinutrino.

$$ {}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\nu}_e $$

β^+ decay ⇒ proton changes into neutron, positron and neutrino.

$$ {}_1p^1 \rightarrow {}_0n^1 + {}_1e^0 + \nu_e $$

Strong nuclearforce > EM force > weak nuclearforce > gravitationalforce.

(high) (Low)

Gravitational force → Long range force.

EM force → infinite range force.

Strong & weak force → Short range forces.

2020-CIVIL, E

A car
two points
was 15 m/sec.
(a) $\frac{1}{3}$ ms

2020-CIVIL, EE

A stone
maximum
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2021-CIVIL

A rubbe
speed of 8 m
High speed
Then the
(a) 16

2021-CIVIL

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horizontal
an angl
(a)

...nature of the Co-Examiner

Name Designation

Y. Veerendra nath, M.Sc

Name, Designation & Address

4. FRICTION

Date :

Experiment No. :

Page No. :

2020-ECE, Mech

The coefficient Kinetic friction μ_k, rolling friction μ_r, static friction μ_s is related by —

✓(a) $\mu_s > \mu_k > \mu_r$ (b) $\mu_s < \mu_k < \mu_r$ (c) $\mu_s < \mu_r < \mu_k$ (d) $\mu_s > \mu_r > \mu_k$.

2020-ECE, Mech

A boy of mass 40 kg is climbing a vertical pole at a constant speed. If the $\mu = 0.8$ and $g = 10 m/s^2$ then the Horizontal force on the pole is —

(a) 300 N (b) 400 N ✓(c) 500 N (d) 600 N.

~~20~~

2020-CIVIL, EEE Identify the correct answer (statement).

(a) static friction depends on area of contact.

(b) Kinetic friction depends on area of contact.

(c) μ_s does not depend on the area of the surface in contact.

✓(d) $\mu_k < \mu_s$.

2020-CIVIL, EEE

The coefficient of friction between the tyres and the road is 0.25. The maximum speed with which a car can be driven round a curve of radius 40 m, without skidding is — ($g = 10 m/s^2$)

$f = \frac{mv^2}{r} \rightarrow \mu N = \frac{mv^2}{r} \rightarrow \mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$

$\rightarrow v = \sqrt{\mu g r} = \sqrt{0.25 \times 10 \times 40}$

$v = \sqrt{\frac{1}{4} \times 400} = \sqrt{100} = 10 m/s.$

2021-CIVIL

Mark the correct statement about the frictional force 'f' when a body slides across a surface with coefficient of friction 'μ'.

(a) The magnitude of 'f' is less than μN

✓(b) 'f' is independent of the area of contact.

(c) 'f' depends on the area of contact.

(d) 'f' is directly proportional to the instantaneous velocity of the body.

$\therefore$ mass → $m \propto v$ . .