

KINETICS

If a force applied by 1st body on 2nd body , then it will moves either Translation , Rotation, Vibration or Oscillation So that motion of one body can be happen due to the effect of second body.

Example: 1) a ball hit by bat. 2) falling apple by earth gravity 3) vehicle moving by force exerted by petrol 4) earth motion by force of sun 5) motion of solar family due to force of galaxy 6) motion of galaxy due to force of universe.....

Our scientists were continuously trying to find the reason for that force exerted by universe.

Imagine a bicycle rider on road moving, by applying a Torque force on pedals continuously. Now the bicycle possesses two types of motion, those are straight line (non directional – without slope) motion and curved (directional – with slope) motion.

Straight line motion is also known as translatory (or) linear motion. Here the Bicycle having linear components.

Curved motion is also known as Rotation (or) oscillatory (or) vibratory motion. Here the Bicycle having Angular momentum components on wheels.

Linear components:

1) Linear displacement (or) distance moved on straight road ----- $\vec{S}$

Units ----- metre , km , feet.....

It is number of perimeters($2\pi r$) moved by tyre on road

2) Linear velocity ----- $\vec{V} = \frac{d\vec{S}}{dt}$, units ---- km/hr , m/sec

It tells , at which rate the distance moved by a vehicle.

3) Linear acceleration ----- $\vec{a} = \frac{d\vec{V}}{dt}$, units ---- m/sec²

It tells , at which rate the velocity changes (may increases or decreases)

4) Linear Momentum ----- $\vec{P} = m \vec{V}$, units ---- kg. m/sec

It tells , the velocity with mass combination (or) dynamic velocity.

5) Impulse ---- $I = \Delta P = F . \Delta t$, units ---- Newton .sec ----- kg. m/sec

It tells , how much breaking force used in some duration of time.

6)Linear (or) Applied Force ---- $\vec{F} = \frac{d\vec{P}}{dt} = m \vec{a}$, units ---- Newton or dyne

It tells , how the momentum of the body changes (increase or decrease)

With in the motion , if Force $F=0$ ---- motion continues with constant momentum. If force $F \neq 0$ --- motion continues with variable momentum .

7)Linear work done $W = \vec{F} \cdot \vec{S} = F S \cos \theta$ by the external applied Force(1st body).

Units ----- joule

8)Linear Potential energy P.E = mgh --- units --- joule

It tells , energy gained by 1st body due to work done by applied force. Energy gained(stored) by its position.... part of work done by 1st body.

9)Linear Kinetic energy K.E = $\frac{1}{2} m v^2$ ----units ---- joule.

It tells , energy gained by 1st body due to external force (work done) by 2nd body. Energy gained in motion..... some part of work done by 1st body.

10)Power $p = \frac{\text{work}}{\text{time}} = \frac{\text{force} \times \text{displacement}}{\text{time}} = \text{force} \times \text{velocity}$ --- joules/sec --- watts

-- it tells , how much energy converted into work (or) vice versa rate of doing work (or) energy flow.

Angular Components:

Imagine a Bicycle wheel , Giant wheel Linear components is along the perimeter , angular components at the centre or at ball barring.

1)Angular Displacement ---- Angle (θ) ---- used to identify the position of an object in circular or rotation or oscillation or vibration motion.

Units ----- radians, degrees.

2)Angular Velocity ---- $\omega = \frac{d\theta}{dt}$ ----- units --- radians/sec

It tells , at which the angle changes with time at centre, rate of change in angular displacement.

3)Angular acceleration ---- $\alpha = \frac{d\omega}{dt}$ ----- units --- radians/sec²

It tells , at which rate the angular velocity changes at centre.

4) Angular momentum ----- $\vec{L} = \vec{p} \times \vec{r} = P r \sin \theta$, units ---- kg. m²/sec²

It tells , the momentum of the body in circular motion.

5) Angular Force (or) Torque ----- $\vec{\tau} = \vec{F} \times \vec{r} = F r \sin \theta$, units --- Newton . metre

It is the force along the radius. Two equal forces in opposite directions on the body to move it as circle.....

It creates the rate of change in angular momentum..... $\vec{\tau} = \frac{d\vec{L}}{dt}$

With in the motion , if external torque $\vec{\tau} = 0$ ---- motion continues with constant angular momentum , if torque $\vec{\tau} \neq 0$ ---- motion continues with variable angular momentum.

6) Moment of inertia -- ----- $I = m r^2$ ----- units ---- kg . meter²

It tells , the mass of the body in rotation

7) Angular work done $W = \vec{F} \cdot \vec{s} = \vec{F} \times 2\pi\vec{r} = 2\pi (\vec{F} \times \vec{r}) = 2\pi \tau$ ----- units ---joule

It tells , work done by torque force (1st body) along radius vector .

8) Angular potential energy --- P.E= no potential energy stored in rotating bodies.

9) Angular kinetic energy --- k.E = $\frac{1}{2} I \omega^2$ ----- units ---- joule

It tells , energy gained due to work done by 1st body due to torque force applied.

10) Angular power ---- $P = \text{Torque} \times \text{angular velocity}$ ----- $P = \tau \omega$ ----- watts

It tells , how much energy will be transferred in rotation.

Relation between linear & Angular components:

Consider a particle or an object moving along circular motion with radius 'r', then

The position of the object can be changes in every second can be identified by angle 'θ' at centre.

If it has complete one rotation , that is ----- $2\pi^R = 360^\circ$ ---- $1^R = \frac{360^\circ}{2\pi} = 57.3^\circ$

Similarly it has linear displacement 'x' on perimeter.

If the angle moved at centre is $\theta = 1^R$ ---- $1 = \frac{x}{r}$ ---- $x = r$ ---- arc = radius

$\theta = 2^R$ ---- $2 = \frac{x}{r}$ ---- $x = 2r$ ---- arc = double radius

It is clear that the arc length (x) is directly proportional to angle 'θ'.

$x \propto \theta$ ---- $x = r \theta$ ---- here 'r' is proportionality constant.

If the position per every second , $\frac{dx}{dt} = r \frac{d\theta}{dt}$ ----- $V = r \omega$

Linear velocity of the particle on perimeter = radius X angular velocity at centre.

If an external force is applied , then the particle velocity may increase or decrease.

This change is $\frac{dv}{dt} = r \frac{d\omega}{dt}$ ----- $a = r \alpha$

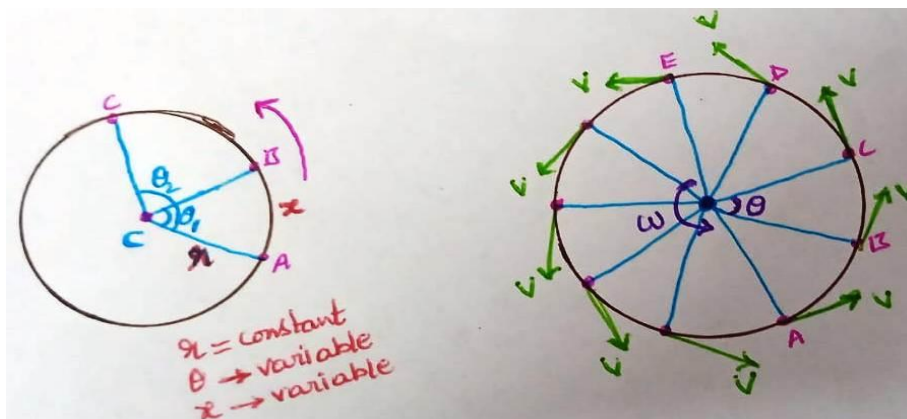
Linear acceleration on perimeter = radius X angular acceleration at centre.

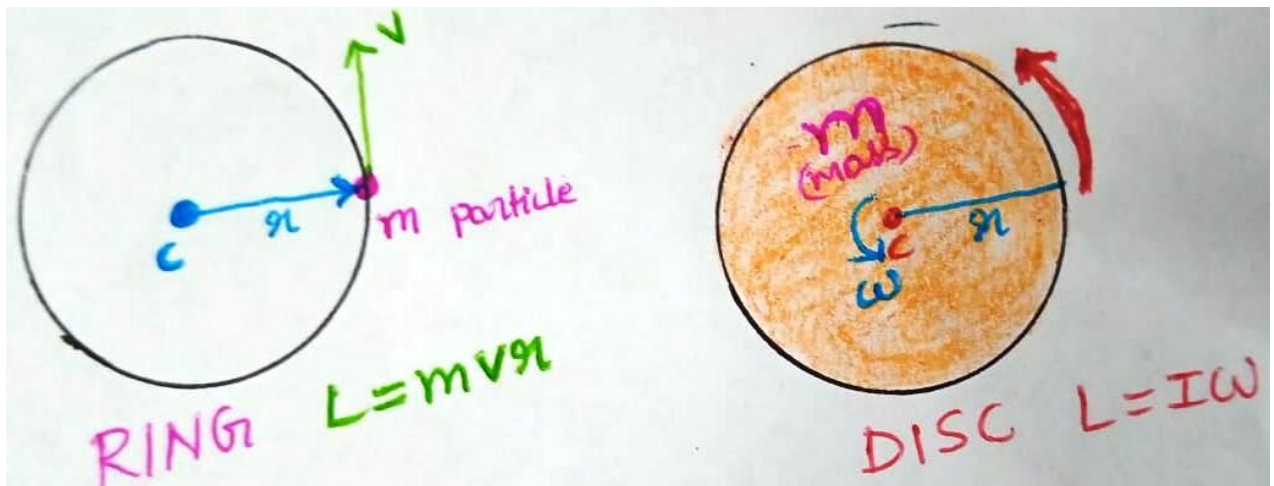
Angular momentum: It is momentum of the body in rotational motion.

$$\vec{L} = \vec{p} \times \vec{r} = mvr$$

But $v = r \omega$ ----- $L = m (r\omega) r = mr^2\omega$

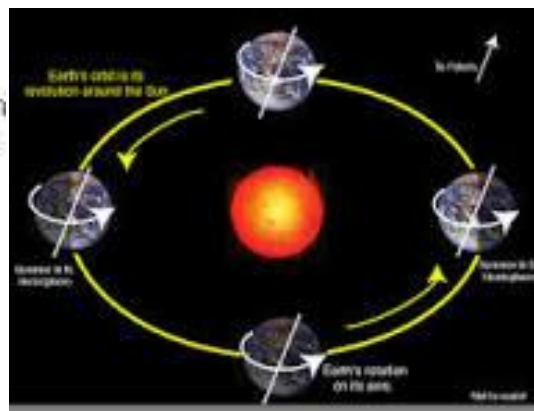
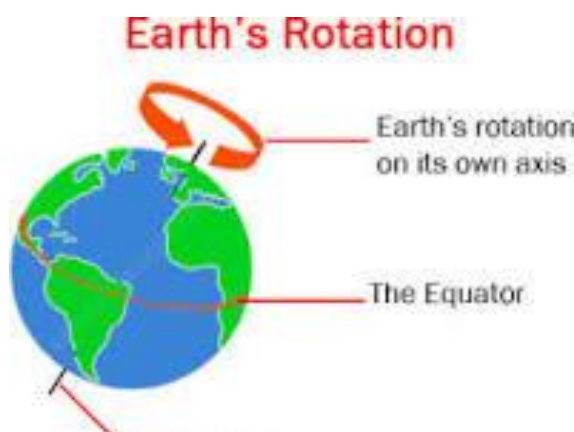
$L = I \omega$ Here $I = mr^2$ is moment of inertia of system





Torque: If we apply a torque (moment of force), it will change the angular momentum of the system, hence an there obtain an angular acceleration(α) at the centre.

That is $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{dL}{dt} = \frac{d(I\omega)}{dt} = I \frac{d\omega}{dt}$ ----- $T = I \alpha$



Kinetic Energy: Hence , total system acquire motional energy .

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m (r\omega)^2 = \frac{1}{2} mr^2 \omega^2$$

$$K.E = \frac{1}{2} I \omega^2 , \text{ here } I = m r^2 \text{ moment of inertia of the system}$$

Some Examples:

- 1) In linear case ----Imagine a CAR and BUS moving with same **linear** momentum (P), then $P = m_1 v_1 = m_2 v_2$ ----- $\frac{v_1}{v_2} = \frac{m_2}{m_1}$ ----- $v \propto \frac{1}{m}$

Linear Velocity of vehicle tyre is inversely proportional to mass,, So that car moving with high velocity than bus.

- 2) In angular case ---- Imagine small fan, large fan are moving with same **angular** momentum(L),

$$\text{Then } L = I_1 \omega_1 = I_2 \omega_2 \text{ ----- } \frac{\omega_1}{\omega_2} = \frac{I_2}{I_1}$$

So, angular velocity at the middle of the fan is inversely proportional to it's moment of inertia(mass in rotation) . So that angular velocity of small fan is higher than that of large fan. Hence small fan moves high frequency and speed.



- 3) Take the ring shaped wheel and disc shaped wheel , and make it turn. Which will come to rest first (or) which will continues rotations?

Answer ---- ring will comes to rest first---- high moment of inertia

Disc will continue some rotations----low moment of inertia



- 4) Imagine train wheels, slowly speed up---- because high moment of inertia at rest ,, in high speed motion ---- moment of inertia decreases,,, So that the pilot will take some time to stop the train .

- 5) why Bicycle crank is large than Bike crank :

Torque (force along circle) is used to move a body in rotation.

$$\text{Torque} = \text{Force} \times \text{radius} = 10 \times 4 = 4 \times 10$$

Large force used on short radius, Small force is used on large radius.

In Bicycle front crank radius is large, back crank radius is high. Hence, small force is enough to ride or move.

In Bike front crank is small, back crank radius is small. Hence, large force is required to move that is given by engine(strokes) in bike.



6) Bohr atomic model 2nd postulate:

The intention of Neil Bohr on Atomic model(1913) is how the electrons are continuously moving around the nucleus.

He said that “ the Angular momentum of electron is conserved”.

It means , angular momentum is constant – motion continuous in some orbital.

$$\vec{\tau} = \frac{d\vec{L}}{dt} , \text{ Torque (force along radius) creates angular momentum.}$$

If the external (further) torque is Zero(0) , no change(increase or decrease) in angular momentum happened. That is angular momentum does not change, it means body moves continuously.

$$\vec{\tau} = 0 \implies \frac{d\vec{L}}{dt} = 0 \implies L = \text{constant} \implies mvr = \text{constant.}$$

Bohr had gave the expression for that constant using debroglie and plank theories.

Debroglie ---- wave and particle duality—moving particle having wavelength. Momentum $P = \frac{h}{\lambda}$

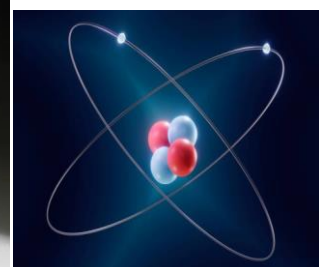
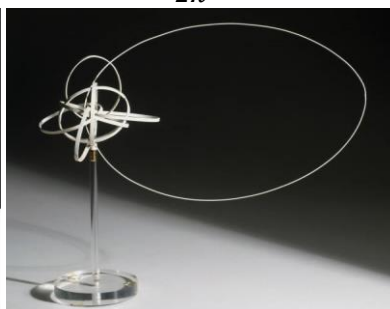
He gave the statement as ---- electron(particle) moving in various circular orbits must possess wavelength.

That is perimeter of orbit = orbit number x wavelength of its wave

$$2\pi r = n\lambda \implies 2\pi r = n \frac{h}{p}$$

$$2\pi r = n \frac{h}{mv}$$

$$L = mvr = \frac{nh}{2\pi} = \text{constant} (n, h, 2, \pi = \text{constant}) \text{-----(1)}$$



velocity of an electron in an orbit is $v = \frac{z}{n} \times 2.18 \times 10^6 \text{ m.s}^{-1}$

here z = atomic number, n = orbit number

so, velocity is inversely to atomic number. From eq (1) velocity and radius are inversely to each other(both in multiplication)..

Hence, radius(r) is directly proportional to atomic number(n).

That is electron in first orbit has high velocity, and velocity decreases with radius increases, but the angular momentum of electron is constant for an orbit. Electrons in various orbits moving with various constant angular momentums.

Finally orbit(2D) is replaced with orbital(3D shape) because of Heisenberg Uncertainty Principle(1927).

“It states that , the momentum($P=mv$) and position(r) of an electron are not

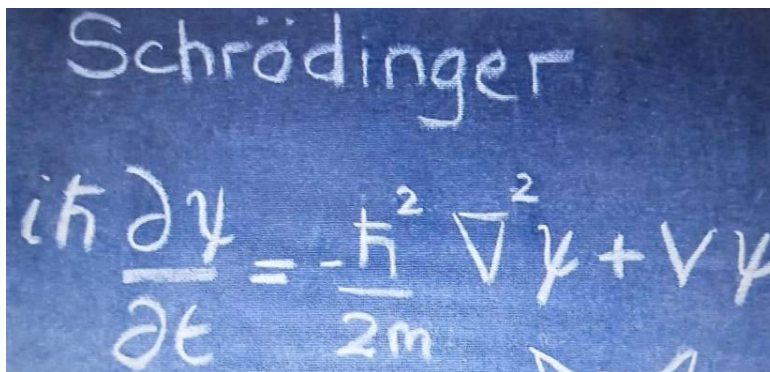
exist at a time”. $\Delta P \cdot \Delta r \geq \frac{h}{4\pi}$ ---- P and r in inversely to each

And energy and time are not exist at a time for an electron in orbit.

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi} \text{ ----- } E \text{ and } t \text{ in inversely to each.}$$

Their product gives angular momentum. So that electrons are actually rotating in orbital shape. Here their velocities, radius, energy, time linear momentum are continuously changes.

Finally the shape of orbital is achieved by Schrödinger wave equation.



Schrödinger

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$


$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

