ELET

1. UNITS - Dimensions — 2 marks

Model-1: Dimensions for given physical quantities. → force → $kg\frac{m}{sec^2}$ → $M^1L^1T^{-2}$ — (50)

Model-2: Homogeneity principle → All terms in a physical equation having same UNITS & dimensions. → maths & science different
$2+3=5$; $2+3=?$; $2kg+3kg=5kg$; $2kg+3sec=?$ not possible — 1st point in science

1) $v=At^2+Bt+C$ here v–velocity, t–time. What are the UNITS of A, B, C.
Homogeneity principle → $\underbrace{V}_{velocity}=\underbrace{At^2}_{velocity}+\underbrace{Bt}_{velocity}+C$ → velocity.
∴ velocity $=At^2 \to A=\frac{v}{t^2} \to \frac{m/sec}{sec^2} \to m/sec^3$
velocity $=Bt \to B=\frac{v}{t} \to m/sec^2$
velocity $=C \to C \to m/sec$.

2) $x=At+Bt^2$, x–distance, t–time. $A=\frac{x}{t} \to m/sec \to$ velocity, $B=\frac{x}{t^2} \to m/sec^2 \to$ acceleration.

3) $\underset{acceleration}{a}=At^2+Bt \implies A=\frac{a}{t^2} \to \frac{m/sec^2}{sec^2} \to m/sec^4$
$B=\frac{a}{t} \to m/sec^3$.

4) vandarwall eq. of gas ⇒ $(P+\frac{a}{V^2})(V-b)=RT$, P → pressure, V → volume, T → temperature, R → constant. a, b = ?
$P+\frac{a}{V^2}$ ⇒ pressure + pressure
∴ $\frac{a}{V^2}$ = pressure ⇒ $a=P\times V^2$
$a=\frac{N}{m^2}\times(\frac{M}{sec})^2 \to kg\frac{\frac{m}{sec^2}\cdot\frac{m^2}{sec^2}}{m^2} \to kg\, m/sec^4$
$V-b$ = volume ⇒ b also volume.

5) $\mu=A+\frac{B}{\lambda^2}$, μ → Refractive index, λ → wavelength.

6) what are the UNITS & dimensions for $S_n=u+a(n-\frac{1}{2})$; u → velocity, a → acceleration, n → time.
so → S_n → velocity → m/sec → $M^0L^1T^{-1}$.

Model-3: To check (or) find the equation.

1) which quantity represents $\sqrt{\frac{P}{d}}$ here P–pressure, d–density.
$\sqrt{P/d} \to \sqrt{\frac{F}{A\cdot d}} \to \sqrt{\frac{kg\, m/s^2}{m^2\; kg/m^3}} = \sqrt{\frac{kg\, m\cdot m^3}{m^2\, kg\, sec^2}} = \sqrt{m^2/sec^2} \to m/sec \to$ velocity.

2) $2\pi\sqrt{\frac{l}{g}}=?$ l–length, g → acceleration → Ans: time — m → massdensity → $\frac{mass}{length}$ → Ans: time.

3) $2l\sqrt{\frac{m}{T}}=?$ l–length, T → Tension (mg), m → massdensity → Ans: velocity.

4) $\sqrt{g\lambda}=?$ g → acceleration, λ → wavelength → Ans: velocity → Ans: acceleration due to gravity.

5) $\frac{GM}{R^2}=?$ M → mass, R → Radius, G → gravitational constant → Ans: velocity orbital.

6) $\sqrt{\frac{2GM}{R}}=?$ M → mass, R → Radius, G → gravitational constant

Model-4: Derive a new physical equation:- find expression for force.

1) force depends on pressure, velocity, time.
$F\propto P^xV^yt^z \implies [M^1L^1T^{-2}]=[M^1L^{-1}T^{-2}]^x\times[M^0L^1T^{-1}]^y\times[M^0L^0T^1]^z$
$x+0+0=1$, $\underline{x=1}$; $-x+y+0=1$, $\underline{y=2}$; $-2x-y+z=-2$, $-2-2+z=-2$, $\underline{z=2}$; ∴ $F\propto pv^2t^2$

2) force $F\propto A^xv^yd^z$ Ans (1,2,1).
3) force $F\propto m^xv^yr^z$ Ans (1,2,1)
4) Time period $T\propto P^xd^yE^z$. Ans $(-\frac{5}{6},\frac{1}{2},\frac{1}{3})$.
5) velocity $V\propto p^xd^y$
6) force $F\propto Y^aS^bV^c$ Ans (0) here Y → young's modulus, S → surface Tension, V → veloci

Model-5: convert 1 system to another.

1) $1\,kg/m^3$ = — gm/cm^3 ⇒ $\frac{1000\,gm}{(100\,cm)^3} \to \frac{10^3}{10^6}gm/cm^3 \to 0.001\,gm/cm^3$.
2) 1 newton = — dyne ⇒ $1\,kg\,m/s^2 \to 1000\,gm\cdot100\,cm/s^2 \to 10^5\,gm\cdot cm/s^2 \to 10^5$ dyn
3) 1 Joule = — erg ⇒ $1\,kg\,m^2/s^2 \to 1000\,gm\cdot(100\,cm)^2/s^2 \to 10^7$ ergs.
4) mercury density $13.6\,gm/cm^3$ convert into SI (MKS) system ⇒ $13.6\times1000\,kg/m^3 \to 13600$

Model-6 :- Midineous.

1) If the unit of length is tripled, mass doubled, time remains same, then what happens to force.

force $\rightarrow$ kg m/sec^2 $\rightarrow M^1L^1T^{-2}$; Now mass — 2M, Length $\rightarrow$ 3L, Time — T

$\therefore$ force $\rightarrow (2M)^1 \times (3L)^1 \times T^{-2} \rightarrow 6M^1L^1T^{-2} \rightarrow 6F \rightarrow$ force increases 6 times.

2) mass $\rightarrow$ 2M, length $\rightarrow$ L/2, time $\rightarrow$ 2T, then pressure? $\Rightarrow P \rightarrow M^1L^{-1}T^{-2} \rightarrow \frac{M^1}{L^1T^2}$

$P \rightarrow \frac{(2M)^1}{(\frac{L}{2})^1(2T)^2} \rightarrow \frac{2M^1}{\frac{L^1}{2}\cdot 4T^2} \rightarrow \frac{M^1}{L^1T^2} \rightarrow P$ $\rightarrow$ unchanged.

3) the product of two quantities P and Q is $M^1L^2T^{-2}$ and division of two quantities $\frac{P}{Q}$ is $M^1L^0T^{-2}$ then what is dimensions of P x Q.

$P \times Q \rightarrow M^1L^2T^{-2}$

$\frac{P}{Q} \rightarrow M^1L^0T^{-2}$

$\Rightarrow (P \times Q) \times \frac{P}{Q} \rightarrow (M^1L^2T^{-2})(M^1L^0T^{-2})$

$P^2 \rightarrow M^2L^2T^{-4} \Rightarrow P = M^1L^1T^{-2} \rightarrow$ force.

and $PQ \rightarrow M^1L^2T^{-2}$

$Q \rightarrow \frac{M^1L^2T^{-2}}{M^1L^1T^{-2}} \rightarrow M^0L^1T^0 \rightarrow$ distance.

3+4 = 7 $\rightarrow$ Maths

3kg + 4sec = NP $\rightarrow$ phy

3kg + 4kg = 7kg

3apples + 4 oranges = 7fruits $\rightarrow$ Psychological

$\rightarrow$ what is the dimensional formula for capacitance $\rightarrow$

$\rightarrow \frac{A}{B} = m$, here A - force, m - Linear mass density, B - ?

$\rightarrow$ What are the dimensions for torque $\rightarrow M^1L^2T^{-2}$

angular momentum $\rightarrow M^1L^2T^{-1}$

$\rightarrow$ "

$\rightarrow$

Same dimension set

1) Impulse, Linear Momentum
2) stress, modulus of elasticity
3) Torque, work, energy, Heat (cal, Joule, erg)
4) pressure, Stress
5) frequency, Angular velocity.
6) force constant, Surface Tension

NO UNITS, NO dimensions

1) $\rightarrow$ Refractive index
2) $\rightarrow$ coefficient of friction
3) $\rightarrow$ Strain

$\sqrt{\frac{1}{10^2}}$

$\frac{10^3 x^2}{2x}$

~~[illegible]~~

$\frac{10}{0.1}$

$\sqrt{\frac{x}{4x}}$

$\sqrt{2}$

$\frac{1}{0.01}$

$\sqrt{0.5^2 - 0.4^2}$ =

$\sqrt{8}$

$\sqrt{3}$ = %

$\sqrt{2}$ =

$\frac{\sqrt{3}}{2}$ = %

1 = %

0 = %

$\frac{1}{\sqrt{2}}$ = %

$\frac{1}{2}$ = %

$\frac{a/b}{c}$ =

$\frac{a}{b/c}$ =

$\frac{a/b}{b/c}$ =

$\frac{x}{x}$ =

$\frac{x}{x}$ =

$\frac{x}{x^2}$ =

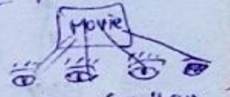
[illegible]

Vectors (2M)

magnitude → value → 20 kg, 40 m/sec, 15 min ...

direction → x, y, z, left, Right, up, down, front, Back, angle (θ) ...

→ 2nd point in Science



θ varies then screen size (Mag) varies

→ i.e no two persons can see a body equally.

Physical quantities

Scalar quantities

→ having only magnitude
(or)
magnitudes varies without direction
(or)
Can not change magnitude with direction.
(or)
which have equal Mag in all directions (ex: time).

Ex:- time, distance, Area, volume, speed, density, work, power, energy, mass (m), current, pressure

Vector quantities

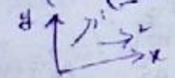
→ having both Mag & direction.
(or)
mag varies with direction
(or)
can change Mag with direction
(or)
having various mag in various directions
(or)
Same Mag but direction changes (Ex:- radius)

Ex:- displacement, plane, velocity, acceleration, force, momentum, Impulse, weight (mg)

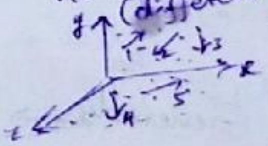
Axial (or) pseudo vectors
→ Angular velocity ω, Angular Momentum $\bar{L}$, Torque $\bar{\tau}$.

Types of vectors

Coplanar vectors (same plane)



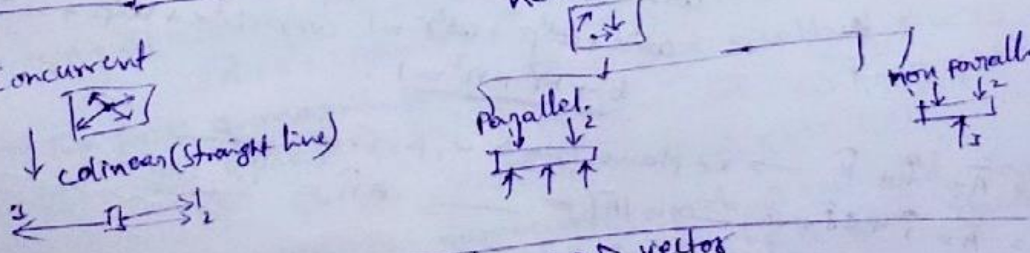
non coplanar (different planes)



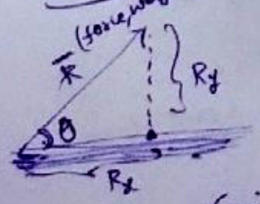
1, 4 → non

Coplanar vectors:

- Concurrent
  ↓ colinear (Straight line)
- non concurrent
  - Parallel
  - non parallel



Resolution of a vectors (in a plane) → 2D vectors



$R_x = R\cos\theta$, $R_y = R\sin\theta$

$\bar{R} = R_x\hat{i} + R_y\hat{j}$

$R = \sqrt{R_x^2 + R_y^2}$, $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

Ex:- A mass of weight 60 kg inclined on a wall with 30°. find Horizontal & vertical weights.

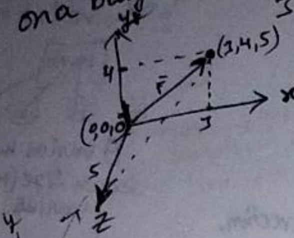
$W_x = W\cos\theta = 60\cos 30° = 30\sqrt{3}$ Kg on (ground)

$W_y = W\sin\theta = 60\sin 30° = 30$ kg on (wall).

Model-1 Notations & ~~Angles~~ Angles.

force on a body $\bar{F} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ —— 5N in z.

$\downarrow$ 3N in x-directin —— 4N in y



Total force with respect to origin $|\bar{F}| = \sqrt{(3-0)^2+(4-0)^2+(5-0)^2}$

$= \sqrt{9+16+25}$

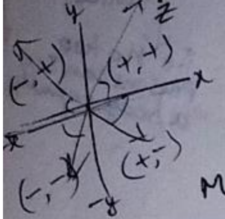
$= \sqrt{50} = 5\sqrt{2}$ Newton

Vector = scalar × direction

$\bar{A} = |\bar{A}|\hat{n}$

direction (or) UNIT vector $\hat{n} = \frac{\bar{A}}{|\bar{A}|} = \frac{\text{vector}}{\text{Magnitude}}$

for above force $\hat{n} = \frac{3\hat{i}+4\hat{j}+5\hat{k}}{5\sqrt{2}} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$.

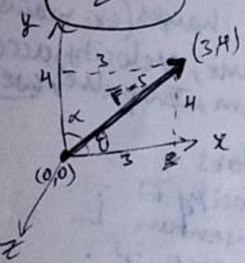


Magnitude on xy plane $\Rightarrow \sqrt{3^2+4^2} = \sqrt{9+16} = 5$ Newton

yz " $\Rightarrow \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$ Newton

zx " $\Rightarrow \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$ Newton.

2D



$\bar{F} = 3\hat{i}+4\hat{j}$ — Angle made with x axis is $\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right) = \sin^{-1}\left(\frac{4}{5}\right) = \cos^{-1}\left(\frac{3}{5}\right)$

" " y-axis $\Rightarrow \alpha = \tan^{-1}\left(\frac{3}{4}\right) = \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$

$\alpha = 90° - \theta = 90° - \tan^{-1}\left(\frac{3}{4}\right)$

$\alpha = \cot^{-1}\left(\frac{4}{3}\right)$. $= \frac{\pi}{2} - \tan^{-1}\left(\frac{3}{4}\right)$

" " z-axis $\Rightarrow \beta = 90°$

Directional cosines :- 3D

Angles represented by Horizontal sides (or) Adjacent sides

that is by 'cos' functions.

not in plane, it is in space (3D).

$\bar{F} = 3\hat{i}+4\hat{j}+5\hat{k}$ ——> not in plane, it is in space (3D).

Angle with x-axis ——> $\cos\alpha = \frac{3}{\sqrt{50}}$ ——> $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{50}}\right)$

" y-axis ——> $\cos\beta = \frac{4}{\sqrt{50}}$ ——> $\beta = \cos^{-1}\left(\frac{4}{\sqrt{50}}\right)$

z-axis ——> $\cos\gamma = \frac{5}{\sqrt{50}} \rightarrow \frac{5}{5\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45°$.

Generally ——> $\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ ——> $|A| = \sqrt{A_x^2+A_y^2+A_z^2}$

$\cos\alpha = \frac{A_x}{|A|}$, $\cos\beta = \frac{A_y}{|A|}$, $\cos\gamma = \frac{A_z}{|A|}$.

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{A_x^2}{A^2} + \frac{A_y^2}{A^2} + \frac{A_z^2}{A^2} = \frac{A_x^2+A_y^2+A_z^2}{A^2} = \frac{A^2}{A^2} = 1$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (or) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$l^2 + m^2 + n^2 = 1$.

——> $\bar{A} = \hat{i} + \hat{k}$ ——> xz plane ——> α, β, γ. = ____

——> $\bar{A} = \hat{i} + 2\hat{j} + \sqrt{2}$ ——> $|\bar{A}|$ = ____, $\hat{An}$ = ____

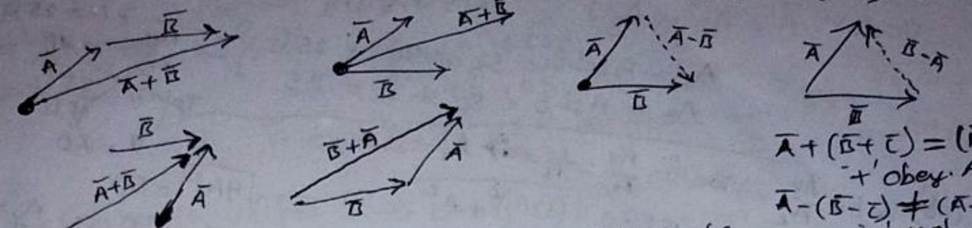
——>

Model - 2 — Addition & Subtraction.

$\bar{A} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\bar{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

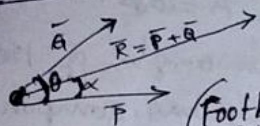
$\bar{A}+\bar{B} = 3\hat{i} - \hat{j} + 7\hat{k}$, $\bar{B}+\bar{A} = 3\hat{i} - \hat{j} + 7\hat{k} \Rightarrow \bar{A}+\bar{B} = \bar{B}+\bar{A}$
'+' obey commutative

$\left.\begin{array}{l}\bar{A}-\bar{B} = -\hat{i} + 5\hat{j} - \hat{k} \\ \bar{B}-\bar{A} = \hat{i} - 5\hat{j} + \hat{k}\end{array}\right\}$ $\left.\begin{array}{l}\bar{A}-\bar{B} \neq \bar{B}-\bar{A} \\ \bar{A}-\bar{B} = -(\bar{B}-\bar{A})\end{array}\right\}$ '−' not commute.



$\bar{A} + (\bar{B}+\bar{C}) = (\bar{A}+\bar{B}) + \bar{C}$
'+' obey Associative
$\bar{A} - (\bar{B}-\bar{C}) \neq (\bar{A}-\bar{B}) - \bar{C}$
'−' not Associative.

Parallelogram law: → to all type energies → [illegible]



Resultant vector $\bar{R} = \bar{P} + \bar{Q}$
" magnitude $|\bar{R}| = R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$
Resultant direction $\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$

(Football example)
Bench moving example

→ $|\bar{A}+\bar{B}|^2 + |\bar{A}-\bar{B}|^2 = \left(\sqrt{A^2+B^2+2AB\cos\theta}\right)^2 + \left(\sqrt{A^2+B^2-2AB\cos\theta}\right)^2$
$= 2A^2 + 2B^2 = 2(A^2+B^2)$

✓ → $|\bar{A}+\bar{B}|^2 - |\bar{A}-\bar{B}|^2 = 4AB\cos\theta$

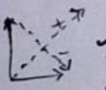
✓ → $P = Q = R \Rightarrow \theta = ?$ $\theta = 120^\circ$, $\alpha = 60^\circ$

✓ → $P = Q$ & $\theta = \theta$ $R = 2\cos(\theta/2)$, $\alpha = \theta/2$ → if $P = Q$, $\theta \to 2\theta \Rightarrow R = 2\cos\theta$.

→ $|\bar{A}+\bar{B}| = n|\bar{A}-\bar{B}|$ ~~$\Rightarrow \theta =$~~ ? and $|\bar{A}| = |\bar{B}|$ then $\theta = ?$
$A^2 + B^2 + 2AB\cos\theta = n^2(A^2 + B^2 - 2AB\cos\theta)$
$2A^2 + 2A^2\cos\theta = n^2(2A^2 - 2A^2\cos\theta)$
$1 + \cos\theta = n^2(1 - \cos\theta)$ → $\frac{\theta}{2} = \tan^{-1}\left(\frac{1}{n}\right) \Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{n}\right)$
$\cancel{2}\cos^2\theta/2 = n^2 \cdot \cancel{2}\sin^2\theta/2$ $\theta = 2\cot^{-1}(n)$
(or) $\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (or) $\theta = \sin^{-1}\left(\frac{2n}{1+n^2}\right)$
(or) $(1+n^2)\cos\theta = n^2 - 1 \Rightarrow$

(or) $\cos^2(\theta/2) = n^2\sin^2(\theta/2)$
$1 - \sin^2(\theta/2) = n^2\sin^2(\theta/2)$
$1 = (n^2+1)\sin^2(\theta/2)$
$\sin\theta/2 = \frac{1}{\sqrt{n^2+1}}$
$\theta = 2\sin^{-1}\left(\frac{1}{\sqrt{n^2+1}}\right)$

✓ → $|\bar{A}-\bar{B}| = n|\bar{A}+\bar{B}|$ and $|\bar{A}| = |\bar{B}|$ then $\theta = ?$ → $\theta = 2\cot^{-1}\left(\frac{1}{n}\right) = 2\tan^{-1}(n)$



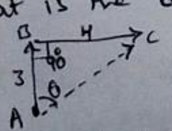
✓ → $|\bar{A}-\bar{B}| = |\bar{A}+\bar{B}|$ then $\theta = ?$ → $n = 1 \Rightarrow \theta = 2\tan^{-1}(1) = 90^\circ$

✓ → $|\bar{A}-\bar{B}| = \sqrt{3}|\bar{A}+\bar{B}|$ then $\theta = ?$ → $n = \sqrt{3} \Rightarrow \theta = 2\tan^{-1}(\sqrt{3}) = 2(60^\circ) = 120^\circ$

→ $|\bar{A}+\bar{B}| = |\bar{C}|$ and $A^2 + B^2 = C^2$ then $\theta = ?$
$\sqrt{A^2+B^2+2AB\cos\theta} = C \Rightarrow \cancel{A^2} + \cancel{B^2} + 2AB\cos\theta = \cancel{C^2} \Rightarrow 2AB\cos\theta = 0 \Rightarrow \theta = 90^\circ$

✓ → Two forces of magnitudes 3:5, $\theta = 60^\circ$ and resultant $F = 35$ find F_1, F_2.
$F_1 = 3x$, $F_2 = 5x$, $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \Rightarrow F_1 = 15$, $F_2 = 25$
Sol: (or) $F_1 = \frac{3}{5}F_2$, $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \Rightarrow F_2 = 25$, $F_1 = 15$.

→ A man waking towards North 3 km and take right turn and then walk 4 km, how far away he is from starting point, What is the direction of this final position.



$\bar{AC} = \bar{AB} + \bar{BC}$
$|\bar{AC}| = \sqrt{3^2 + 4^2 + (2 \times 3 \times 4 \cos 90^\circ)} = \sqrt{9 + 16} = 5$ km.
$\theta = \tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}(1.33) = 53^\circ$.

→ Three vectors of magnitudes 8, 16, 20 are in same plane. What are min & max values of resultant?
Min → $20 - 16 \to 4$, & $8 - 4 = 4$ UNITS
Max → $20 + 16 + 8 \to 44$ UNITS.

→ If $\bar{C}$ is added to $\bar{B}$ the result is $-9\hat{i} - 8\hat{j}$ and if $\bar{B}$ is subtracted from $\bar{C}$ the result is $5\hat{i} + 4\hat{j}$. What is the direction of $\bar{B}$.
$\bar{B} + \bar{C} = -9\hat{i} - 8\hat{j}$
$\bar{C} - \bar{B} = 5\hat{i} + 4\hat{j}$ $\Rightarrow 2\bar{B} = -14\hat{i} - 12\hat{j}$
$\bar{B} = -7\hat{i} - 6\hat{j}$ → $(3)^{rd}$ coordinate
$\theta = \tan^{-1}\left(\frac{6}{7}\right) =$

→ A vector $\bar{A}$ of magnitude 50 m. is in xy plane & makes an angle of 30° with x-axis. What are the x, y components, what is the vectors.

vector $\bar{A} = A_x\hat{i} + A_y\hat{j}$, $|\bar{A}| = \sqrt{A_x^2 + A_y^2} = 50$

$A_x = A\cos\theta = 50\cos 30^\circ = 25\sqrt{3}$

$A_y = A\sin\theta = 50 \cdot \sin 30^\circ = 25$

$\rightarrow \bar{A} = 25\sqrt{3}\hat{i} + 25\hat{j}$

$\frac{A_y}{A_x} = \tan\theta$

$\tan\theta = \frac{25}{25\sqrt{3}}$

$\theta = 30$

(or) $\tan\theta = \frac{A_y}{A_x} \Rightarrow \tan 30^\circ = \frac{A_y}{A_x} = \frac{1}{\sqrt{3}} \Rightarrow A_x = \sqrt{3}A_y$.

$\sqrt{A_x^2 + A_y^2} = |\bar{A}| \Rightarrow \sqrt{(\sqrt{3}A_y)^2 + A_y^2} = 50 \Rightarrow \sqrt{4A_y^2} = 50$

$\Rightarrow 2A_y = 50 \Rightarrow A_y = 25$

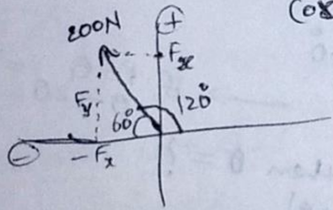
$\therefore A_x = 25\sqrt{3}$

$\therefore \bar{A} = 25\sqrt{3}\hat{i} + 25\hat{j}$

→ A force of 200 N is applied on a body at an angle of 120° with the x-axis. What are the rectangular components.

Horiz $F_x = F\cos\theta = 200\cos 120^\circ = 200 \times \frac{-1}{2} = -100\,N$

$F_y = F\sin\theta = 200\sin 120^\circ = 200 \times \frac{\sqrt{3}}{2} = 100\sqrt{3}$ N.



(or) $F_x = -200\cos 60^\circ$

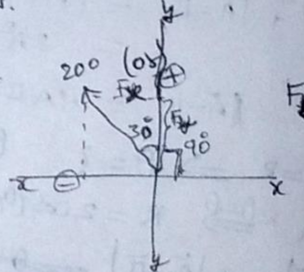
$= -200 \times \frac{1}{2}$

$= -100$ N

$F_y = 200\sin 60^\circ$

$= 200 \times \frac{\sqrt{3}}{2}$

$= 100\sqrt{3}$ N.



$F_y = 200\cos 30^\circ$

$= 200 \times \frac{\sqrt{3}}{2}$

$= 100\sqrt{3}$

$F_x = -200\sin 30^\circ$

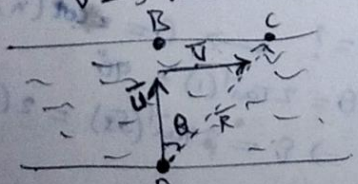
$= -200 \times \frac{1}{2}$

$= -100$ N

→ Triangle Law :- Application (Boat in a River).

U → Boat velocity

V → water velocity.

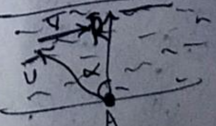


$\overline{AC} = \overline{AB} + \overline{BC}$

$\bar{R} = \bar{U} + \bar{V}$

$\theta = \tan^{-1}\left(\frac{V}{U}\right)$

Rowing.



$\bar{R} = \bar{U} + \bar{V}$

$\alpha = \sin^{-1}\left(\frac{V}{U}\right)$

Model (3)

Multiplication of vectors :-

① Scalar with vector → Answer is Another vector.

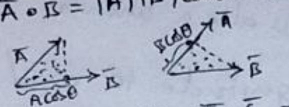
Ex:- force $\bar{F} = m\bar{a}$ = mass × acceleration
(vector) (scalar × vector).

Momentum $\bar{P} = m\bar{V}$
Impulse $I = Ft$

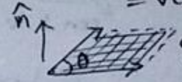
$\bar{F} = 2\,kg \times (6\hat{i} + 2\hat{j} + 3\hat{k})\ m/sec^2$
$= 12\hat{i} + 4\hat{j} + 6\hat{k}\ kg\ m/sec^2$
$\bar{F} = 12\hat{i} + 4\hat{j} + 6\hat{k}$ Newtons. → $|\bar{F}| = \sqrt{144+16+36}$
$= \sqrt{196}$ newtons.

② vector with vector → Answer ① scalar ② vector.

DOT (or) scalar product
$\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}|\cos\theta$ → Magnitude only (scalar)



CROSS (or) vector product.
$\bar{A} \times \bar{B} = |\bar{A}||\bar{B}|\sin\theta\ \hat{n}$ → direction.
Magnitude
= vector.



gives Area & Rotation in perpendicular direction.

$\hat{k}\cdot\hat{k} = \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1$
$\hat{i}\cdot\hat{j} = 0$
$\hat{j}\cdot\hat{k} = 0$
$\hat{k}\cdot\hat{i} = 0$

Ex:- work $W = \bar{F}\cdot\bar{S} = FS\cos\theta$
PE $= mgh$
Power $P = \frac{W}{t} = \frac{\bar{F}\cdot\bar{S}}{t} = \bar{F}\cdot\bar{V}$
flux $\phi = \bar{B}\cdot\bar{A}$

$\hat{i}\times\hat{i} = 0$
$\hat{j}\times\hat{j} = 0$
$\hat{k}\times\hat{k} = 0$
$\hat{i}\times\hat{j} = \hat{k}$ (-)
$\hat{j}\times\hat{k} = \hat{i}$ (-)
$\hat{k}\times\hat{i} = \hat{j}$ (-)

Ex:- linear velocity $\bar{v} = \bar{r}\times\bar{\omega}$
Angular Momentum $\bar{L} = \bar{r}\times\bar{P}$
Torque $\bar{\tau} = \bar{F}\times\bar{r}$
Bolt Tightning, cyclon, screw driver.

$\bar{A}\cdot\bar{B} = \sqrt{A_x^2+A_y^2+A_z^2} \times \sqrt{B_x^2+B_y^2+B_z^2} \times \cos\theta$

$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \Sigma\,\hat{i}(A_xB_y - A_yB_x)$.

$\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z$.
to find angle $\theta = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|}\right)$

Model (3)

→ 20 N force acts on a body with $\theta = 60°$ and produces displacement 10 m. find work.? $W = \bar{F}\cdot\bar{S} = FS\cos 60° = 20\times10\times\cos60° = 200\times\frac{1}{2} = 100$ Jole

→ find work done by the force $\bar{F} = 3\hat{i} - 2\hat{j} + 3\hat{k}$ displaces displacement $\bar{S} = 3\hat{i} - 2\hat{j} - 2\hat{k}$, $W = \bar{F}\cdot\bar{S} = (3,-2,3)\cdot(3,-2,-2) = 9+4-6 = 7$ Joules.

✓→ find the power delivered by a body using force $\bar{F} = (6,2,-1)$ to displace a displacement $\bar{S} = (2,2,1)$ in 3 secs.
$P = \frac{W}{t} = \frac{\bar{F}\cdot\bar{S}}{t} = \frac{(6,2,-1)\cdot(2,2,1)}{3} = \frac{12+4-1}{3} = \frac{15}{3} = 5$ watts.

✓→ Two vectors $\bar{A} = (2,x,3)$, $\bar{B} = (1,-1,2)$ are ⊥ to eachother then $x = ?$
$\theta = 90°$ → $\bar{A}\cdot\bar{B} = 0 \Rightarrow (2,x,3)\cdot(1,-1,2) = 0 \Rightarrow 2 - x + 6 = 0 \Rightarrow x = 8$.

✓→ Find the angle between $\bar{A} = (2,2,4)$, $\bar{B} = (3,-1,2)$.
$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta \Rightarrow (2,2,4)\cdot(3,-1,2) = \sqrt{4+4+16}\ \sqrt{9+1+4}\ \cos\theta$
$6 - 2 + 8 = \sqrt{24}\ \sqrt{14}\ \cos\theta$
$\cos\theta = \frac{12}{\sqrt{24}\sqrt{14}} = \frac{\sqrt{12}}{\sqrt{2}\sqrt{2}\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \Rightarrow \theta = \cos^{-1}\left(\sqrt{\frac{3}{7}}\right)$
(or) $\sqrt{1-\sin^2\theta} = \sqrt{\frac{3}{7}} \Rightarrow \theta = \sin^{-1}\left(\frac{4}{\sqrt{7}}\right)$
$\Rightarrow \theta = \tan^{-1}\left(\sqrt{\frac{4}{3}}\right)$.

✓→ What is the angle between two similar vectors.
$\bar{A}\cdot\bar{A} = |\bar{A}||\bar{A}|\cos\theta \Rightarrow (A_x^2+A_y^2+A_z^2) = \sqrt{A_x^2+A_y^2+A_z^2} \times \sqrt{A_x^2+A_y^2+A_z^2}\ \cos\theta$
$\Rightarrow 1 = \cos\theta \Rightarrow \theta = 0°$

→ if $\bar{A}\cdot\bar{B} = nAB$ then
$\theta = \cos^{-1}(n)$
$\theta = \sin^{-1}(\sqrt{1-n^2})$
$\theta = \tan^{-1}\left(\frac{\sqrt{1-n^2}}{n}\right)$

✓→ if $\bar{A}\cdot\bar{B} = \frac{\sqrt{3}}{2}AB$ then what is angle ?
$\bar{A}\cdot\bar{B} = AB\cos\theta \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$

→ Angle between face diagonals of a simple cube of each side 'a'. →

→ Angle between face diagonals of a cube of sides a, b, c θ →

→ Angle between two body diagonals of a simple cube of each side 'a' θ →

→ " " sides a, b, c → θ =

→ " one face diagonal & body diagonal of simple cube of each side a θ →

→ " sides a, b, c θ →

$\rightarrow \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ — parallel (or) similar vectors

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0 \rightarrow$ perpendicular vectors

$\rightarrow \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$\hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{k} = \hat{i}, \quad \hat{k}\times\hat{i} = \hat{j}$ $\qquad \hat{i}\cdot(\hat{j}\times\hat{k}) = 1$

$\hat{j}\times\hat{i} = -\hat{k}, \quad \hat{k}\times\hat{j} = -\hat{i}, \quad \hat{i}\times\hat{k} = -\hat{j}$

$\rightarrow$ find the $\perp$ vector for $3\hat{i}+4\hat{j}$

$\bar{A}\cdot\bar{B} = 0 \Rightarrow (3\hat{i}+4\hat{j})\cdot(4\hat{i}-3\hat{j}) = 0$

$12-12=0$ ✓

So ~~there~~ is ∞ answers.

$\Rightarrow n\hat{k}$ is also a answers.

$\rightarrow$ find a vector having a magnitude 10 and perpendicular to $3\hat{i}-4\hat{j}$.

$\bar{A}\cdot\bar{B} = 0 \Rightarrow (3\hat{i}-4\hat{j})\cdot(4\hat{i}+3\hat{j}) = 0 \longrightarrow 12-12$ ✓

magnitude $\Rightarrow \sqrt{16+9} = 5$

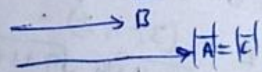
$\therefore |n(4\hat{i}+3\hat{j})| = 10$

$n\sqrt{16+9} = 10 \Rightarrow n = 2$

$\therefore$ the vector $\bar{B} = n(4\hat{i}+3\hat{j}) = 2(4\hat{i}+3\hat{j}) = 8\hat{i}+6\hat{j}$.

(or) $\pm 10\hat{k}$ is also a vector (answer)

$\rightarrow$ Find a vector having same magnitude of $\bar{A} = 6\hat{i}-8\hat{j}$ and parallel to $\bar{B} = 3\hat{i}+4\hat{j}$



$|\bar{C}| = |\bar{A}|$

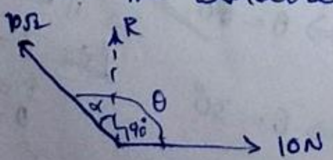
$|n\bar{B}| = |\bar{A}| \Rightarrow |n(3\hat{i}+4\hat{j})| = |6\hat{i}-8\hat{j}|$

$\Rightarrow n\sqrt{9+16} = \sqrt{36+64}$

$\Rightarrow n\times 5 = 10 \Rightarrow n = 2$

$\therefore$ required vector $\bar{C} = 2\bar{B} = 6\hat{i}+8\hat{j}$

$\rightarrow$ The resultant of two forces 10 N & $10\sqrt{2}$ N makes 90° with the smaller force. Find the angle between them.



$\theta = 90^\circ + \alpha$

$\bar{A} = 10\hat{i} + 0\hat{j}$

$\bar{B} = 10\sqrt{2}\cos\alpha\,\hat{i} + 10\sqrt{2}\sin\alpha\,\hat{j}$

$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta$

$100\sqrt{2}\cos\alpha + 0 = 10\times 10\sqrt{2}\times\cos\theta$

$\cos\alpha = \cos\theta$

$\cos\alpha = \cos(90^\circ+\alpha)$

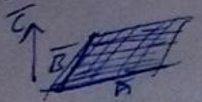
$\cos\alpha = \sin\alpha \Rightarrow \alpha = 45^\circ$

$\therefore \theta = 90^\circ + 45^\circ = 135^\circ$.

Model ④: Cross product

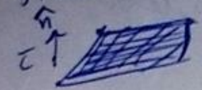
$\bar{A}\times\bar{B} \neq \bar{B}\times\bar{A}$ } not obeys commutative
$\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$ } same magnitude, but directions

$\rightarrow$ $\bar{A} = (1,2,1)$ $\bar{B} = (2,1,3)$ vectors forms a parallelogram, find Area



$$\bar{C} = |\bar{A}\times\bar{B}| = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \hat{i}(6-1) - \hat{j}(3-2) + \hat{k}(1-4)$$
$$= 5\hat{i} - \hat{j} - 3\hat{k}$$
$$\text{Area} = \sqrt{25+1+9} = \sqrt{35}$$

$\rightarrow$ find a UNIT vector forming by parallelogram by $\bar{A} = (1,2,1)$, $\bar{B} = (2,3,1)$



$$\hat{n} = \frac{\bar{C}}{|\bar{C}|} = \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|}$$
$$\bar{A}\times\bar{B} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = \hat{i}(2-3) - \hat{j}(1-2) + \hat{k}(3-4)$$
$$= -\hat{i} + \hat{j} - \hat{k}$$
$$\hat{n} = \frac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{1+1+1}} = \frac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}.$$

$\rightarrow$ find Tork produced by $\bar{F} = (2,3,4)$ along a radius vector $\bar{r} = (2,1,3)$.

$\bar{T} = \bar{F}\times\bar{r}$ (or) $\bar{r}\times\bar{F}$

$\rightarrow$ linear velocity $\bar{V} = \bar{r}\times\bar{\omega}$ $\rightarrow$ radius vector $\times$ Angular velocity

$\rightarrow$ Angular momentum $\bar{L} = \bar{r}\times\bar{P}$

Model ⑤

✓ $\rightarrow$ $|\bar{A}\cdot\bar{B}| = |\bar{A}\times\bar{B}| \Rightarrow \theta = ?$ $\Rightarrow$ $AB\cos\theta = AB\sin\theta$ $\rightarrow$ $\theta = 45°$

$\rightarrow$ $|\bar{A}\cdot\bar{B}| = n|\bar{A}\times\bar{B}| \Rightarrow \theta = ?$ $\Rightarrow$ $AB\cos\theta = n\,AB\sin\theta \Rightarrow \theta = \tan^{-1}\left(\frac{1}{n}\right) = \cot^{-1}(n)$
$\theta = \sin^{-1}(\ )$
$\theta = \cos^{-1}(\ )$

✓ $\rightarrow$ $|\bar{A}\times\bar{B}| = n|\bar{A}\cdot\bar{B}| \Rightarrow \theta = ?$

✓ $\rightarrow$ $|\bar{A}\cdot\bar{B}|^2 + |\bar{A}\times\bar{B}|^2 = ?$ $\Rightarrow A^2B^2$

$\rightarrow$ Scalar product & vector product of two vectors are $48\sqrt{3}$ and 144 then $\theta = ?$

$AB\cos\theta = 48\sqrt{3}$ $\Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$ $\Rightarrow \theta = 60°$.
$AB\sin\theta = 144$

$\rightarrow$ $\hat{i}\cdot(\hat{j}\times\hat{k}) = ?$ $\Rightarrow \hat{i}\cdot\hat{i} = 1$.

$\rightarrow$ $(\hat{j}\times\hat{k})\cdot(\hat{i}\times\hat{j}) = ?$ $\Rightarrow \hat{i}\cdot\hat{k} = 0$

Model ③

$\rightarrow$ Three vectors $\bar{A}, \bar{B}, \bar{C}$ satisfy the relation $\bar{A}\cdot\bar{B} = 0$ and $\bar{A}\cdot\bar{C} = 0$. Then vector $\bar{A}$ is parallel to

(a) $\bar{B}$ (b) $\bar{C}$ (c) $\bar{B}\cdot\bar{C}$ (d) $\bar{B}\times\bar{C}$

$\underbrace{\bar{B}\cdot\bar{C}}_{\text{scalar, not vector.}}$

$\bar{A}\cdot\bar{B} = 0$ $\quad$ $\bar{A}\cdot\bar{C} = 0$
$AB\cos\theta = 0$ $\quad$ $AC\cos\alpha = 0$
$\theta = 90°$ $\quad$ $\alpha = 90°$

$\bar{B}\times\bar{C}$ is along $\bar{A}$ $\Rightarrow$ i.e parallel

✓ Model (2)

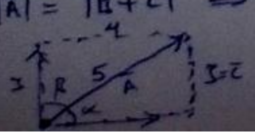
$\rightarrow$ if $\bar{A} = \bar{B}+\bar{C}$ and the magnitudes of $\bar{A}, \bar{B}, \bar{C}$ are $5, 4, 3$ then the angle between $\bar{A}$ & $\bar{C}$ is ——

$\bar{A}$ is the resultant of $\bar{B}, \bar{C}$.

given $\bar{A} = \bar{B}+\bar{C}$ $\Rightarrow$

$|\bar{A}| = |\bar{B}+\bar{C}|$ $\Rightarrow$ $5 = \sqrt{4^2+3^2+2\cdot4\cdot3\cos\theta}$ $\Rightarrow \theta = 90°$

so $\bar{B}, \bar{C}$ are $\perp$ to each.



$\beta = \tan^{-1}\left(\frac{3}{4}\right) = \sin^{-1}\left(\frac{4}{5}\right) = \cos^{-1}\left(\frac{3}{5}\right)$

model (2) The add & Sub vectors of two vectors having magnitude $\sqrt{a^2+b^2}$, what is angle between add & sub?

$|\bar{A}| \rightarrow a+b$
$|\bar{B}| \rightarrow a-b$
$|\bar{R}| = \sqrt{a^2+b^2}$

$\bar{R} = \bar{A} + \bar{B}$

$|\bar{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\sqrt{a^2+b^2} = \sqrt{(a+b)^2 + (a-b)^2 + 2(a^2-b^2)\cos\theta}$

$a^2+b^2 = 2a^2+2b^2+(2a^2-2b^2)\cos\theta$

$-a^2-b^2 = (2a^2-2b^2)\cos\theta$

$\cos\theta = \dfrac{-(a^2+b^2)}{2(a^2-b^2)}$

Model (2) if $|\bar{A}| = |\bar{B}|$ and $|\bar{A}| = |\bar{A}| - |\bar{B}|$ then what is the angle between $\bar{A}, \bar{B}$.

$\sqrt{A^2} = \sqrt{A^2 - 2\bar{A}\cdot\bar{B} + B^2}$

$A^2 = A^2 + B^2 - 2\bar{A}\cdot\bar{B} \Rightarrow 2\bar{A}\cdot\bar{B} = B^2$

$\Rightarrow 2AA\cos\theta = A^2 \quad \because |\bar{A}| = |\bar{B}|$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$ //.

2014 → The magnitude of the resultant of $A+B$ and $A-B$ is
(1) $2A$ (2) $\sqrt{A^2+B^2}$ (3) $2B$ (4) $\sqrt{A^2-B^2}$

→ given $\bar{A}\cdot\bar{B} = 0$ and $\bar{A}\times\bar{C} = 0$, the angle between $\bar{B}$ and $\bar{C}$ is
135°, 90°, 180°, 45°

→

$$\begin{array}{r} 20.2 \\ 9.8 \\ \hline 10.4 \end{array}$$

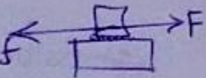
Friction

Opposing force between two surfaces in contact

Laws → 1) it is independent on Area

2) dependent of nature of Surfaces
↓ Rough, Smooth, oil (lubricants)

3) $f \propto N \rightarrow \boxed{f = \mu N}$



$\mu \rightarrow$ coefficient of friction

$0 \le \mu \le 1$

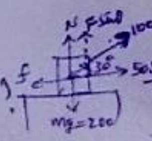
$N = mg \rightarrow$ Horizontal surface

$N = mg\cos\theta \rightarrow$ on inclined plane

$\mu_s > \mu_k > \mu_r$

Static (hight), Kinetic, Rolling (low).

Ex:- 1) A block of weight 200N is pulled with a force 100N with 30° angle. $\mu = ?$

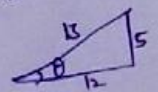


~~$f = 50\sqrt{3}$, $N = 200$~~

$f = F \Rightarrow f = \mu N$, $F\cos\theta = \mu(mg - F\sin\theta) \Rightarrow \mu = 0.574$

2) for inclined plane $\mu = \tan\theta$, $\theta \rightarrow$ Repose angle

3) A body slides down on a Rough inclined plane that rises 5 in 13. What is the coefficient of friction.



$\sin\theta = \frac{5}{13} \Rightarrow \tan\theta = \frac{5}{12}$

$\mu = \frac{5}{12} = 0.4$

4) A body slides down on inclined plane of angle 30°. find the velocity after 3secs. $\mu = 0.4$.

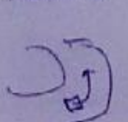
$v = u + at$

$v = 0 + g(\sin\theta - \mu\cos\theta)t \Rightarrow v = 9.8(\sin 30° - 0.4\cos 30°) \times 3$

$v = 9.8\left(\frac{1}{2} - 0.4 \times \frac{\sqrt{3}}{2}\right) \times 3$

$= 4.52$ m/sec.

5) What is the Max speed of car on a curved path of radius 'r' with coefficient of friction μ.



$F = f$

$\frac{mv^2}{r} = f \Rightarrow \frac{mv^2}{r} = \mu N$

$\Rightarrow \frac{mv^2}{r} = \mu mg$

$\Rightarrow v = \sqrt{\mu g r}$

and $v = r\omega$

$\omega = \frac{v}{r} = \frac{\sqrt{\mu g r}}{r}$

$\omega = \sqrt{\frac{\mu g}{r}}$ Angular speed

(6) Bicycle tyre ⇒ friction ⇒ front wheel → back friction
Rear wheal → front side Friction

(7) A block of mass M is lying on a horizontal frictionless surface. one end of a rope mass m is fixed to the block and a force F is applied at the free end parallel to the surface. The force acting on the block ——

$\frac{FM}{M-m}$ $\quad \frac{FM}{M+m}$ $\quad \frac{Fm}{M+M}$ $\quad F$

6) the min stoping distance for a car of mass m, moving with a speed v along a level road, if μ is there.

$f = F \Rightarrow \mu mg = ma \Rightarrow a = \mu g$

$v^2 - u^2 = 2as \Rightarrow v^2 - 0 = 2\mu g s$

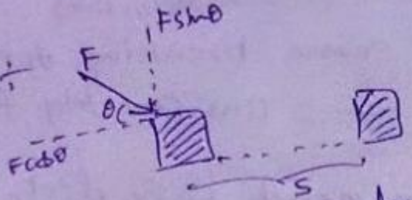
$S = \frac{v^2}{2\mu g}$

Work - power - energy (3) → with reason

Momentum $P = mV$, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

force $F = ma$

Work ÷



work $W = FS\cos\theta$ → $\theta = 0° \to W = FS$ max, $\theta = 90° \to W = FS(0) = 0$, $\theta = 180° \to W = -FS$.

UNITS → Joules → $kg\,m^2/s^2$ → MKS (or) SI system

erg → $gm\,cm^2/s^2$ → CGS system.

kg. metre → gravitational MKS unit

gm. cm → " CGS "

$1\,\text{Joule} = 10^7\,\text{ergs}$

Types ÷

1) workdone by gravity (or) position (or) height ⇒ $P.E = W = mgh$
2) " in motion ⇒ $K.E = W = \frac{1}{2}mv^2$
3) " on inclined plane ⇒ $W = mgh\sin\theta$
4) " in pulling a pendulum ⇒ $W = mgl(1-\cos\theta)$
5) " in expanding (or) compressing a spring $W = \frac{1}{2}Kx^2$, $K = \frac{F}{x}$ force constant
6) " in pulling a Rod of length 'l' is $W = \frac{mgl}{2}(1-\cos\theta)$
7) " due to friction $W = fS = \mu NS = \mu mgS$.

Power ÷ Rate of doing work (or) energy

power $= \frac{\text{work}}{\text{time}}$ → Joule/sec → watts, $1\,hp = 746$ watts

1) power on inclined plane $P = f \times V = mgV\sin\theta$
2) power of gun $P = \frac{n\frac{1}{2}mv^2}{t}$, $n \to$ no. of bullets
3) Power of Motor (or) pump $P = \frac{mgh}{t}$
4) if a body having both height & velocity then $P = \frac{P.E + K.E}{t}$
5) Power of heart $= 1.199$ watts → 1.2 Joule/sec.

Work-energy theorem

Workdone by a force is equal to change in K.E

$W = F \times S = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ Ex ÷ $m = 5\,kg$, $u = 4\,m/s$, $v = 8\,m/s$, $W = ?$ 120 J

$W = \Delta K.E \to W = F \times S$

Energy conservation ÷

1) Energy can't be destroyed & created
2) can change from one form to another form.

ie the total energy of the body is constant.

Ex ÷ water cycle, Bike - petrol - Motion, bulb, Fan - - - -

NOTE ÷ 1) $K.E = \frac{1}{2}mv^2$ ⇒ if velocity → $2V$, mass → $\frac{m}{2}$

then $KE = \frac{1}{2} \cdot \frac{m}{2} \cdot 4v^2 \to 2 \cdot \frac{1}{2}mv^2 \to 2\,K.E$ doubled

2) if velocity increases 50% then $K.E = ?$

$K.E = \frac{1}{2}m\left(v + \frac{v}{2}\right)^2 = \frac{9}{4} \cdot \frac{1}{2}mv^2 = \frac{9}{4}K.E = 2.25\,KE$ final

1.25 times increases

125% more.

3) $K.E = \frac{1}{2}mv^2 \times \frac{m}{m} \Rightarrow \boxed{K.E = \frac{p^2}{2m}}$

take 1 body ⇒ m = constant ⇒ $K.E \propto p^2$

take 2 bodies → if both have same K.E ⇒ $P \propto \sqrt{m}$ heavy mass having more Momen

$W \propto \frac{1}{m} \Rightarrow F.S \propto \frac{1}{M} \Rightarrow$ Take 2 bodies → car, Bus

if both drivers applying same breaking force

F = constant $\Rightarrow S \propto \frac{1}{m}$ → Bus can stop first

→ car, bus can reach same distance.

$F.S \propto \frac{1}{M} \Rightarrow$ if car, bus can reach same distance $F \propto \frac{1}{m} \Rightarrow$ car can apply large force to stop.

S = constant

NOTE:

→ A gun fires 240 bullets per minute. Each bullet of mass 2 gm having velocity 600 m/s. Find the power of gun

$$P = \frac{n \frac{1}{2} M v^2}{t} = \frac{240 \times \frac{1}{2} \times 2 \times 10^{-3} \times (600)^2}{60} = 4 \times 10^{-3} \times 36 \times 10^4$$

$$= 1440 \text{ watts.}$$

→ A Motor pump can lift 2500 litre (or) kg. water per hour from well of depth 30 m.. find power if 25% energy is losses.

$$P = \frac{W}{t} \Rightarrow P = \frac{mgh}{t} \Rightarrow 75\% \, P = \frac{2500 \times 9.8 \times 30}{3600}$$

$$\Rightarrow P = \frac{25 \times 9.8 \times 30}{36} \times \frac{100}{75} = \frac{9800}{36}$$

$P =$ watts

→ if momentum increased by 20% the K.E ——

$P \to P + 20\% P$

$P \to \frac{6P}{5}$

$K.E = P^2/2m$

$$K.E = \left(\frac{6P}{5}\right)^2 / 2m = \frac{36 P^2/2m}{25} = 1.44 \, P^2/2m$$

$$K.E = (1 + 0.44) P^2 = \frac{P^2 + 44\% P^2}{2m}$$

∴ K.E increases 44%

→ N bullets each of mass 'm' kg are fired with a velocity V, at the rate of n bullets per second, upon a wall. The reaction offered by the wall to the bullets is.

(nNmV)

nNV/m

NmV/n

nNm/V

$\cancel{P = \frac{W}{t}}$ $\cancel{F \times V = \frac{\frac{1}{2}mv^2}{t}}$

$\cancel{F \times V = \frac{N (Nm) V^2}{t}}$

$\cancel{F = \frac{N}{t} NmV}$

$F = \cancel{nNmV}$

→ A machine gun fires a bullet of mass 40 gm with a velocity of 1200 m/s. The man holding it can exert a max force of 144 N on the gun. The no. of bullets he can fires per second is

(3)

(4)

(1)

(8)

check

$F = ma = n \cdot \frac{mv}{t}$

$n = \frac{F \times t}{mv}$

$= \frac{144 \times 1}{40 \times 10^{-3} \times 1200} = 3$

$\frac{N}{t} = n = \frac{144}{48} = 3$

$\cancel{P = \frac{W}{t}}$

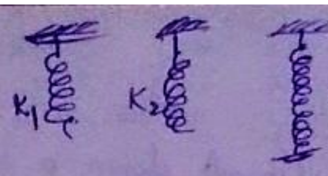
$\cancel{F \times V = \frac{1}{2} \frac{mv^2}{t}}$

$\Rightarrow \cancel{n = \frac{Ft}{mv}} = \cancel{\frac{2 \times 144}{40 \times 10^{-3} \times 1200}} = \cancel{\frac{144}{24}}$

→ under the action of a constant force, a particle experiencing a constant acceleration, then the power (1) zero (2) +ve (3) −ve (4) ↑ with time.

$$\Rightarrow P = \frac{W}{t} = FV = Fat \Rightarrow P \propto t$$

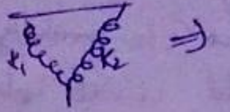
NOTE :- 1) series Spring



$\Rightarrow k = \frac{k_1 k_2}{k_1+k_2}$

$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$.

2) parallel springs.



$\Rightarrow k = k_1 + k_2$.

3) If spring is cut into 2 parts, each ~~two~~ part have 2K (doubles) Spring constant.

Collisions :-

→ A car without passengers moving with certain velocity on a travel ground can be stopped in a distance of 10m.. If the passengers add 25% of its weight, its stopping distance for the same breaking force is ______.

car → one body ⟶ $K.E = \frac{p^2}{m} \Rightarrow W = \frac{p^2}{m}$

$\Rightarrow F \cdot S = \frac{p^2}{m}$

Same F $\Rightarrow S \propto \frac{1}{m}$

Now mass ⟶ $m + 25\% . m \longrightarrow \frac{5m}{4}$

Then distance $S \longrightarrow \frac{1}{\frac{5m}{4}} = \frac{1}{\frac{5 \times 10}{4}}$

$S =$

(a) 15
(b) 10
(c) 7.5
(d) 12.5

→ ~~The minimum stopping distance for a car of mass m, moving with a speed v along a level road, if the force is μ.~~

~~$W = K.E = \frac{1}{2}mv^2$~~

~~$F \times S = \frac{1}{2}mv^2 \Rightarrow \mu N S = \frac{1}{2}mv^2$~~

~~$\Rightarrow \mu m g S = \frac{1}{2}mv^2$~~

~~$\Rightarrow S = \frac{v^2}{2\mu g}$~~

already in friction Lesson

→ ~~Under the action of a constant force, a particle is experiencing a constant acceleration, then the power is~~

(a) zero (b) +ve (c) −ve (d) ↑ uniformly with time.

→ A horizontal force F pulls a 20 kg box at a constant speed along a horizontal floor. If the coefficient of friction between the box and the floor is 0.25. The work done by the force F in moving the box through a distance of 2m.. ______

49 J 196 J 147 J 98 J

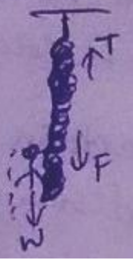
$W = \mu m g S$

→ A uniform rod of mass m and length l is made to stand vertically on one end. The P.E of the rod in this position is ______

$mgl/4$

$P.E = \frac{mgl}{2}(1 - \cos\theta)$

y wants to climb down a rope. The rope can withstand
imum tension equal to two-thired of the weight of the bo
is acceleration due to gravity, the min acceleration with
ich the boy should climb down the rope is.——



Tension $T = \frac{2}{3} mg$

motion $\Rightarrow$ $W - F = T$

$mg - ma = \frac{2}{3} mg \Rightarrow a = g - \frac{2}{3} g$

$a = g/3$

Sound → Medium depended, ~~et~~ Longitudinal waves

Solids > Liquids > gases
high ... low

$V = \sqrt{\frac{Y}{d}}$ Young, density — Granite $v = 6000$ m/s

$V = \sqrt{\frac{B}{d}}$ — Bulk — water $V = 1400$ m/s

$V = \sqrt{\frac{\gamma P}{d}}$ pressure → voice changing

air $V = 320$ m/s $0°c$ $= 340$ m/s $30°$

Sound

Audible Sound to ear
$n = 20$ Hz to 20000 Hz
$\lambda = \frac{v}{n} = \frac{340}{20}$ to $\frac{340}{20000}$
$\lambda = 17$ m to 17 cm.

Inaudible Sound to ear
$n < 20$ Hz — Infrasonics
$n > 20000$ Hz — Ultrasonics → bats, Ants, elephants, Dolphine - ---

Musical sounds
pleasant to ear → talking, ~~ant~~ flute

Noise
Unpleasant to ear → Bike Horn, Hammer Hitting - - - -
highest — Aircraft Noise → 140 dB
Talking — 20 to 50 dB

Noise effects → headache, ~~hbp~~ hbp, Kidney, liver damage, Nerves System collapse, Mental disorder

~~Minim~~ Sound characteristics :-
1) Loudness → Lion (Amplitude)
2) pitch → Honeybea, ~~Ant~~ Musketo (frequency)
3) quality — Comparision of voice

→ Noise level $= 10 \log_{10}^{I/I_0}$, $I_0 = 10^{-12}$ weber/m²

→ Beats :- Superimpose of two nearly equal ~~f~~ sound waves.
Beat frequency $\Delta n = n_1 - n_2 = \frac{V}{\lambda_1} - \frac{V}{\lambda_2}$

Min beats to ear is 10
∴ time $t = \frac{1}{10} = 0.1$ sec

Ex :- Two ~~tun~~ Tunning forks with wave length 5m & 6m produces 10 beats/sec. Find speed of sound in Air
$10 = \frac{V}{5} - \frac{V}{6} \Rightarrow 10 = \frac{6V - 5V}{30} \Rightarrow 10 = \frac{V}{30} \Rightarrow V = 300$

Echo :-
A reflected Sound from an obstacle.
sound velocity $c = \frac{2d}{t} \Rightarrow d = 17$ or 16.5 m → min distance

Applications
1) to find depth of the sea
2) " distance of enemy aircraft
3) " identify stones in Kidney, tumors
4) Megaphone, Stethascope work on this.

Reverberation :- Multiple reflections of sound

Sabine's formula :- $T = \frac{0.16\,V}{\Sigma(as)}$

A man fires a gun and hearing an echo after 2sec. He walks 85 m away from there and hears echo after 2.5 sec. What is the velocity of sound.

$c = \frac{2d}{t} \Rightarrow c = \frac{2d}{2} \Rightarrow c = d$

$c = \frac{2(d+85)}{2.5} \Rightarrow c = \frac{2(c+85)}{2.5} \Rightarrow 2.5c = 2c + 170$

$0.5c = 170 \Rightarrow c = 340$ m/s

Doppler effect:

Change in frequency (pitch) of sound when source of sound and observer are in ~~st~~ relat[ive motion]

Source of sound and observer are in relative motion (observer).

① Source moving towards Stationary lisener (observer).

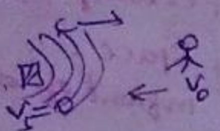
$\Rightarrow n' = \left(\frac{c}{c - v_s}\right) n \qquad n' > n$ sound increases.

② Source Away.

$\Rightarrow n' = \left(\frac{c}{c+v_s}\right) n \qquad n' \downarrow$

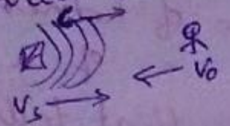
③ Source at Rest, Lisener moving towards Source.

$n' = \left(\frac{c+v_0}{c}\right) n \;,\; n' \uparrow \; \lambda \downarrow$



④ Approach each other

$n' = \left(\frac{c+v_0}{c-v_s}\right) n \;, \quad n' \uparrow$



→ The velocity of sound in air is 330 m/s. The apparant frequency of sound increased by 50% and if source moves towards stationary observer then its velocity is ______

$c = 330$ m/s
$v_0 = 0$
$v_s = ?$
$n' = n + 50\% n$
$n' = \frac{3}{2} n$

$n' = \left(\frac{c}{c - v_s}\right) n$

$\frac{3}{2} n = \left(\frac{330}{330 - v_s}\right) n \Rightarrow 330 - v_s = 330 \times \frac{2}{3}$

$330 - v_s = 220$

$v_s = 110$ m/s

→ An observer moves towards a stationary source of sound with a velocity one tenth the velocity of sound. The apparent increase in frequency ______

$v_s = 0$
$v_0 = \frac{c}{10}$
$n' = ?$

$n' = \left(\frac{c+v_0}{c}\right) n = \left(\frac{c + c/10}{c}\right) n$

$n' = \frac{11}{10} n = 1.1 n \Rightarrow n + 10\% n$

10% increases.