Kinematics & friction (6M)

Motion of the body without Reason (or) cause

without mass, force, momentum, work, energy ---

equations of Motion

1) $v = u + at$
2) $S = ut + \frac{1}{2}at^2$
3) $v^2 - u^2 = 2aS$
4) $S_n = u + a(n - \frac{1}{2})$

t → time
u → initial velocity
v → final "
a → acceleration
S → total distance (scalar)
s → displacement (vector)
S_n → distance travelled in n^{th} sec.

$v = \frac{ds}{dt}$

→ speed (scalar) $= \frac{\text{distance}}{\text{time}}$ → m/sec → Km/hour

$1\,\text{Km/hr} = \frac{5}{18}\,\text{m/sec}$

$54\,\text{Km/h} = 15\,\text{m/s}$

$72\,\text{Km/h} = 20\,\text{m/s}$

→ velocity (vector) $= \frac{\text{displacement}}{\text{time}}$ ⇒ m/sec ⇒ Km/hour

→ acceleration (vector) $= \frac{\text{velocity}}{\text{time}}$ ⇒

$a = \frac{dv}{dt}$ → $\frac{m/sec}{sec}$ → m/sec^2

→ acceleration due to gravity $g = 9.8\,m/sec^2$ → vertical case

g → high → poles
Low → equator
zero → earth center.
decreases → with height, depth

→ for freely falling body $a = +g = +9.8\,m/sec^2$ (Uniform)

$a = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{9.8 - 0}{1 - 0} = \frac{19.6 - 9.8}{2 - 1}$

$= \frac{29.4 - 19.6}{3 - 2}$ - - - -

i.e velocity gradually ↑ i.e changes

9.8, 9.8, 9.8

→ for vertically thrown up body $a = -g = -9.8\,m/sec^2$ (Uniform)

$a = \frac{dv}{dt} = \frac{20.2 - 30}{2 - 1} = \frac{10.4 - 20.2}{3 - 2}$ - - - -

i.e velocity gradually ↓ i.e changes.

→ if uniform velocity $v = u$ ⇒ $a = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v_1 - v_1}{t_2 - t_1}$ ⇒ $a = \frac{0}{dt} = 0$

$v_2 = v_1 \Rightarrow 30\,\text{Km/h} = 30\,\text{Km/h}$

→ if non uniform velocity $v \neq u$ ⇒ $a \neq 0$ ⇒ $a > 0$ (+) or $a < 0$ (−).

$v_2 \neq v_1 \neq v_3$ - - -

→ ~~freely thrown~~

→ Body with frictional Surface ⇒ $a = -\mu g$

μ → coefficient of friction
μ → [0, 1]

if $\mu = 0.5$ ⇒ $a = -0.5 \times 9.8$

$a = -4.9\,m/s^2$

→ Body on inclined plane.

$a = g(\sin\theta - \mu\cos\theta)$, a↓, Body on Rough & moving down

$a = -g(\sin\theta + \mu\cos\theta)$, a↑, Body on Rough & up

$a = g\sin\theta$, a↓, on Smooth & Down

$a = -g\sin\theta$, a↑, on Smooth & up.

→ free falling

$U = 0$

$a = +g = +9.8$

$V = \sqrt{2gh}$ $(v^2 - u^2 = 2as)$

$v = gt$

$t = \sqrt{\frac{2h}{g}}$ $(s = ut + \frac{1}{2}at^2)$

$S_n = g(n - \frac{1}{2})$

vertical thrown up

$V = 0$

$U = U$

$V = 0$

$a = -g = -9.8$

Height travelled $H = \frac{U^2}{2g}$

(or) $U = \sqrt{2gH}$

time $t = \frac{U}{g}$

$S_n = U - g(n - \frac{1}{2})$.

$H_{max} = U^2/2g$

$t_a = t_d = \frac{U}{g}$

$T = \frac{2U}{g}$.

→ A body freely falling from rest has a velocity 'v' after it falls through a distance 'h'. The distance it has to fall down further, for its velocity to become doubled is →

$U = 0$, h, v, h_2, $2v$

$v^2 - u^2 = 2as$

$v^2 - 0^2 = 2gh$

$v = \sqrt{2gh}$

$(2v)^2 - 0^2 = 2g(h_2 + h)$

$4v^2 = 2g(h_2 + h)$

$4 \times 2gh = 2g(h_2 + h)$

$4h = h_2 + h$

$h_2 = 3h$

→ (or) $(2v)^2 - v^2 = 2gh_2$

$4v^2 - v^2 = 2gh_2$

$3v^2 = 2gh_2$

$3 \times 2gh = 2gh_2$

$h_2 = 3h$.

→ An object falls freely from rest for 5 sec. Find the distance travelled by the object in the last 2 sec. $g = 10\,m/s^2$

in 5 secs → $S = \cancel{ut} + \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125$ m.

in 3 secs → $S = \cancel{ut} + \frac{1}{2} \times 10 \times 3^2 = 45$ m.

∴ in 2 secs → $S_5 - S_3 \Rightarrow 125 - 45 = 80$ m.

5 sec, 3 sec, 2 sec.

(or) Last second → 5th one } Last 2 secs.
before last → 4th one

distance travelled in 5th second → $S_5 = U + a(n - \frac{1}{2})$
$= 0 + g(5 - \frac{1}{2})$
$= 10 \times \frac{9}{2} = 45$ m.

→

" " 4th second → $S_4 = 0 + 10(4 - \frac{1}{2})$
$= 10 \times \frac{7}{2} = 35$ m.

in last 2 seconds → 80 m.

→ If a body covers 1st half of the distance with a speed v_1 and 2nd half with speed v_2 then average speed $= \frac{2v_1v_2}{v_1 + v_2}$

→ If a body travels with speed v_1 in time t_1 and with speed v_2 in time t_2 the average speed $= \frac{v_1t_1 + v_2t_2}{v_1 + v_2}$.

→ A car covers the 1st half of the distance at a speed of 40 km/h and the 2nd half at 60 km/h. What is average speed?

(or) $V_{ave} = \frac{2v_1v_2}{v_1 + v_2} = \frac{2 \times 60 \times 40}{100} = 48$ km/h

Time to cover 1st half $t_1 = \frac{x/2}{40} = \frac{x}{80}$

2nd half $t_2 = \frac{x/2}{60} = \frac{x}{120}$

$t = t_1 + t_2 = \frac{x}{80} + \frac{x}{120} = \frac{x}{48}$

Speed $= \frac{\text{distance}}{\text{time}} = \frac{x}{x/48} = 48$ km/h

A freely falling body travels a distance 'x' in the nth second. In the next second if it travels a distance 'y', then

(a) $x + y = g$ (b) $x - y = g$ (c) $y - x = g$ (d) $x = y$

$x = u + g(n - \frac{1}{2})$

$y = u + g(n + 1 - \frac{1}{2})$

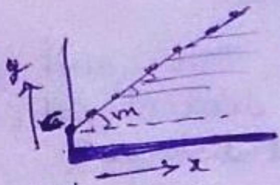
$y = u + g(n - \frac{1}{2}) + g$

$y = x + g$

$y - x = g$ → (c)

Graphs :-

Velocity time graph:-



✓ $v = u + at \rightarrow v = f(t)$

$y = mx + c \rightarrow y = f(x)$

slope $m = \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{y\text{-coordinate}}{x\text{-coordinate}}$

$m = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v - u}{t} = a$

slope (maths) = acceleration (physics)

∴ Slope of velocity – time graph is acceleration.

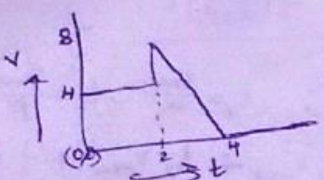
Area under the graph = y-coord × x-coord
$= a \times t$
= velocity → (physics)

Area under graph (maths)

Slope of displacement – time graph is → velocity

Area under velocity – time graph is → displacement.

→ find the distance travelled by the body?



Area under the curve A = y-coord × x-coord

A = velocity × time

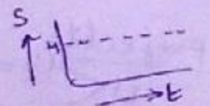
= □ + △

$= (4 \times 2) + (\frac{1}{2} \times 2 \times 8)$

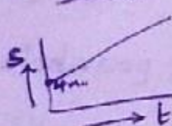
$= 8 + 8$

$= 16$ mts.

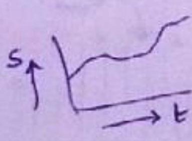
distance = 16 mts.

→ 

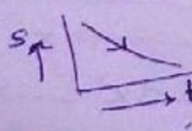
Body at Rest (till now) ie $V = 0 \Rightarrow \frac{ds}{dt} = 0 \Rightarrow S$ = constant



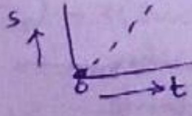
Start moving at 4m, continue distance & with uniform velocity.



Moving with non uniform (discontinued) velocity. ↑ & ↓



Moving with non uniform (decreasing) velocity.

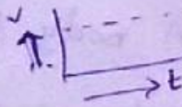


Start from Rest and moving with uniform velocity.

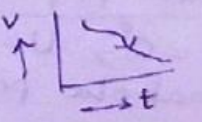
$V = \frac{ds}{dt}$ = constant

$\int ds = \text{const} \int dt$

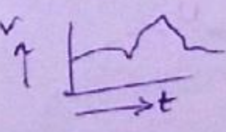
$s = ut + \frac{1}{2}at^2$



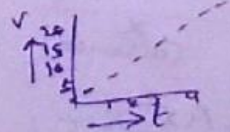
moving with uniform velocity $v = u$, $a = 0$



with deacceleration (or) velocity decreases.



with non uniform velocity



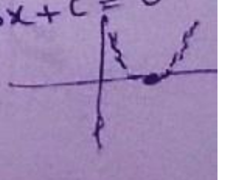
Starting from Rest ($u = 0$) and moving with uniform acceleration

$a = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{5 - 0}{1 - 0} = 5$

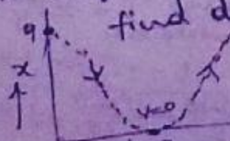
$= \frac{10 - 5}{2 - 1} = 5$

$= \frac{15 - 10}{3 - 2} = 5$

→ distance $S = ut + \frac{1}{2}at^2 \rightarrow \frac{1}{2}at^2 + ut - s = 0$ like $ax^2 + bx + c = 0$



→ Displacement of a particle is $\sqrt{x} = t - 3$, find displacement when its velocity is zero.



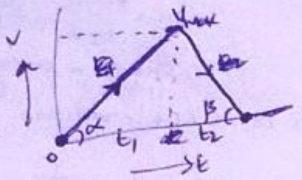
$\sqrt{x} = t - 3$

$x = t^2 + 9 - 6t$

velocity $V = \frac{dx}{dt} = 2t - 6 \Rightarrow V = 0 \Rightarrow t = 3$

$\therefore t = 3 \Rightarrow x = (3)^2 + 9 - 6(3) = 9 + 9 - 18 = 0$

→ A car moving from rest, and getting acceleration 'α' in time t_1 and getting deacceleration β in time t_2. what are the max velocity and total distance?



$\alpha = \frac{V_{max}}{t_1}$, $\beta = \frac{V_{max}}{t_2}$

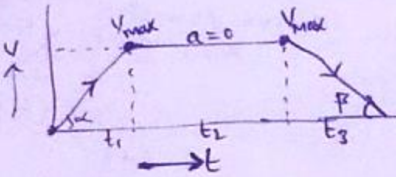
$$t_1 + t_2 = \frac{V_{max}}{\alpha} + \frac{V_{max}}{\beta}$$

$$t = V_{max}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$V_{max} = \frac{t}{\frac{1}{\alpha} + \frac{1}{\beta}} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$$

total distance travelled = velocity × time
= y × x → Area under the graph
$= \frac{1}{2}(t_1+t_2) \times V_{max}$
$= \frac{1}{2} \times t \times \left(\frac{\alpha\beta}{\alpha+\beta}\right) \times t$

$$S = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$$

→ 

$V_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$; $t_1 = \frac{V_{max}}{\alpha}$, $t_2 = \frac{V_{max}}{\beta}$

distance $= \frac{1}{2}t_1 V_{max} + t_2 V_{max} + \frac{1}{2}t_3 V_{max}$

$= \frac{1}{2}\alpha t_1^2 + \alpha t_1 t_2 + \frac{1}{2}\beta t_3^2$

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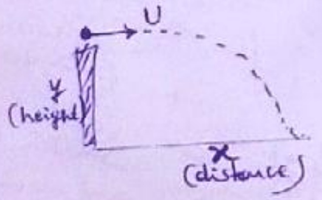
projectile:- A body thrown up other than 90° with the horizontal surface.

Horizontal projectile

$\theta = 0°$, $U = U$

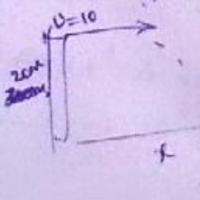
in Parabola path



$y = Ax^2$

$\boxed{y = \left(\frac{g}{2U^2}\right)x^2}$, time $t = \frac{x}{U}$

Parabola eqn.

Ex:- 

$g = 10 m/s^2$
when & where.

$x = \sqrt{\frac{2yU^2}{g}} = \sqrt{\frac{2 \times 20 \times 100}{10}}$

$x = \sqrt{400} = 20m$

$t = \frac{x}{U} = \frac{20}{10} = 2 sec.$

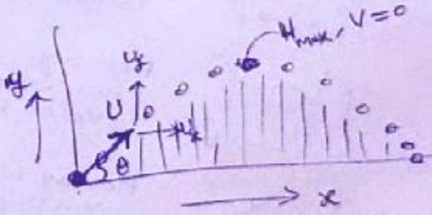
A large no. of bullets are fired in all direction with the same speed u. The maximum area on the ground on which these bullets will spread is.

(a) $\frac{\pi u^2}{g^2}$ (b) $\frac{\pi u^4}{g^2}$ (c) $\frac{\pi u^2}{g^4}$ (d) $\frac{\pi u}{g^4}$

Oblique projectile

$\theta = \theta$, $U \begin{cases} U_x = U\cos\theta \\ U_y = U\sin\theta \end{cases}$

path is parabola.



$y = Ax - Bx^2$ → parabola

$y = (\tan\theta)x - \left(\frac{g}{2U^2\cos^2\theta}\right)x^2$

~~final velocity time t~~

find velocity

$V = \sqrt{U^2 + g^2t^2 - 2Ugt\cos\theta}$

$H_{max} = \frac{U^2\sin^2\theta}{2g}$

Time of Ascent $t_a = \frac{U\sin\theta}{g}$

descent $t_d = \frac{U\sin\theta}{g}$

Flight $T = \frac{2U\sin\theta}{g}$

Range $R = \frac{U^2\sin 2\theta}{g}$
Horizontal distance

$\theta = 45°$ for Max Range $\frac{U^2}{g}$

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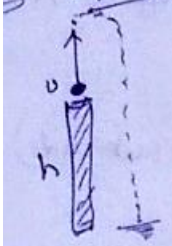
(1) $R = H_{max} \Rightarrow \frac{U^2\sin 2\theta}{g} = \frac{U^2\sin^2\theta}{2g} \Rightarrow \theta = \tan^{-1}(4) = 75.96$

(2) $U = 19.6 m/s$, $\theta = 30° \Rightarrow H_{max} = \frac{19.6^2 \times \frac{1}{4}}{2 \times 9.8} = \frac{19.6}{4} = 4.9 m$

$R = \frac{19.6^2 \cdot \frac{\sqrt{3}}{2}}{9.8} = 19.6\sqrt{3}$.

(3) $R = 200$, $H = ?$

→ Top of the Tower.



height of the tower $h = -ut + \frac{1}{2}gt^2$

① $h = 19.6$, $u = 14.7$ m/s. $t = ?$

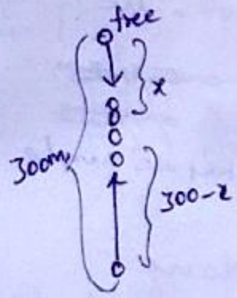
$\cancel{49}$ $19.6 = -14.7t + 4.9t^2$

$t^2 - 3t - 4 = 0 \Rightarrow t = 4$ sec.

~~② Balloon problem.~~

② $h = 39.2$ m., $u = 9.8$ m/s, $t = ?$

③ A stone is fall freely from the top of tower 300 m. hight and at the same time another stone is projected vertically upwards with velocity 75 m/s when and where there will meet.



Stone① → $u = 0$, $s = x$, $a = +g$ → $s = \cancel{ut} + \frac{1}{2}at^2$

$x = \frac{1}{2}gt^2$

Stone② → $u = 75$, $s = 300 - x$, $a = -g$

$300 - x = 75t - \frac{1}{2}gt^2$

$300 - \cancel{x} = 75t - \cancel{x} \Rightarrow t = \frac{300}{75} = 4$ sec.

$t = 4 \longrightarrow x = \frac{1}{2}g(4)^2 = 9.8 \times 8 = 78.4$

meet at $300 - x \longrightarrow 300 - 78.4$

$\longrightarrow 221.6$ m.

~~Friction~~

SHM, Acoustics (sound) [5M]

Motion of a body

- Translatory — Straight line
- Rotatory — about an axis — earth, $\bar{e}$
- Vibratory (or) oscillation — about a fixed point.

Ex:- cell phone vibration
Heart beat pulses
Pendulum, Tuning fork
Sewing machine need.

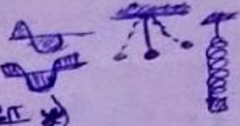
SHM

SHM conditions

1) Motion is to and fro about a fixed point (Position) $\longrightarrow$ up & down, Left & Right, North & South - - - - -
Periodic.

2) $a \propto -x$.

SHM characteristics



1) Displacement (Position) $\longrightarrow$ on y-axis $y = A \sin(\omega t \pm \phi)$

$x = A\cos(\omega t \pm \phi)$

$A\cos(\omega t \pm Kx)$

on x-axis $x = A\cos\left(\frac{2\pi}{T}t \pm \frac{2\pi}{\lambda}x\right)$

$A \rightarrow$ Amplitude $\rightarrow$ Max displacement
$\phi \rightarrow$ phase angle (epoch)
$\omega \rightarrow \frac{\theta}{t} \rightarrow$ angular velocity
$t \rightarrow$ time $\rightarrow$ variable
$K \rightarrow$ propagation constant
$\lambda \rightarrow$ wavelength.

2) velocity $\longrightarrow v = \frac{dy}{dt} \Rightarrow v = A\omega\cos(\omega t \pm \phi) \longrightarrow 90^\circ$ leads to displacement
v leads x by 90°

$\boxed{v = \omega\sqrt{A^2 - y^2}}$

$y = 0$ mean position $\longrightarrow v = \omega A \longrightarrow$ Maximum
$y = \pm A$ extreme $\longrightarrow v = 0$

3) acceleration $\longrightarrow a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t \pm \phi) \longrightarrow$ x & a opposite.

$y = 0 \rightarrow a = 0$
$y = \pm A \rightarrow a = \mp\omega^2 A \longrightarrow$ Max

$\boxed{a = -\omega^2 y} \Rightarrow a \propto -y$

$\frac{a}{y} = -\omega^2 =$ constant $\Rightarrow \frac{\text{acceln}}{\text{displacement}} =$ constant (Angular velocity).

4) Time period $\longrightarrow$ Time taken for one cycle, oscillation, vibration.

$\boxed{T = \frac{2\pi}{\omega}}$ UNITS $\rightarrow$ sec

5) frequency $\longrightarrow$ no. of cycles or vibrations in 1 sec.

$n \rightarrow$ cycles/sec $\Rightarrow$ Hz

$\boxed{n = \frac{1}{T}} \Rightarrow$

$\omega = \frac{2\pi}{T} = 2\pi n$

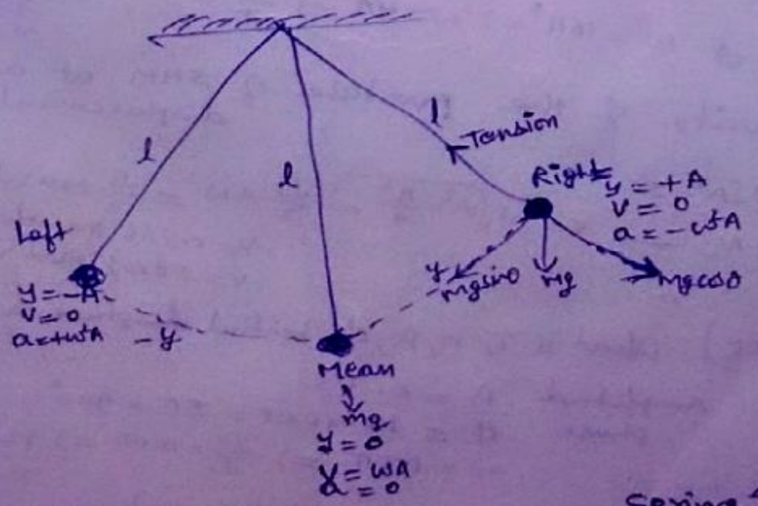
$v = n\lambda$

$a = -\omega^2 y \Rightarrow$ only Magnitude $\longrightarrow a = \omega^2 y \Rightarrow \omega = \sqrt{\frac{a}{y}}$

$\frac{2\pi}{T} = \sqrt{\frac{a}{y}}$

Time $T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceln}}}$ $\quad T \propto \sqrt{y}$

frequency $n = \frac{1}{2\pi}\sqrt{\frac{a}{y}}$, $n \propto \frac{1}{\sqrt{y}}$



Simple pendulum

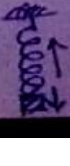
$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$, $T \propto \frac{1}{\sqrt{g}}$

$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \Rightarrow n \propto \sqrt{g}$, $n \propto \frac{1}{\sqrt{l}}$

$g = 4\pi^2 \cdot \frac{l}{T^2}$

1) T is independent of mass, size, material
2) T is independent of Amplitude
3) T dependent on $\sqrt{l}$.

Spring



$T = 2\pi\sqrt{\frac{m}{K}}$ $\quad T \propto \frac{1}{\sqrt{K}}$ $\quad$ Spring constant $K = \frac{F}{x}$

$n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

Bike

Second's pendulum :- A pendulum of time period 2 sec.

$T = 2$ sec. $\Rightarrow T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow 2 = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \frac{g}{\pi^2} = 99.39$ cm

$l \begin{cases} = 0.9939 \text{ m} \\ = 1\text{m} \\ = 100\text{cm} \end{cases}$

$\rightarrow g_{up} = g\left(1 - \frac{2d}{R}\right)$

$\rightarrow g_{down} = g\left(1 - \frac{d}{R}\right)$ $R \rightarrow$ earth radius $R = 6400$ km

$\rightarrow$ pendulum in lift moving up $T = 2\pi\sqrt{\frac{l}{g+a}}$, $T\downarrow, n\uparrow$

$\rightarrow$ " " " down $T = 2\pi\sqrt{\frac{l}{g-a}}$, $T\uparrow, n\downarrow$

$\rightarrow$ freely falling body $a = +g \Rightarrow T = 2\pi\sqrt{\frac{l}{g-g}}$

$T = \infty$

$n = 0 \rightarrow$ does not oscillate.

K.E $\Rightarrow$ $K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t \pm \phi)$

$K.E = \frac{1}{2}m\omega^2(A^2 - x^2)$

mean position $x = 0 \Rightarrow K.E = \frac{1}{2}m\omega^2A^2$

extreme position $x = \pm A \Rightarrow K.E = 0$

P.E $\Rightarrow$ $P.E = \frac{1}{2}Kx^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t \pm \phi)$

$= \frac{1}{2}m\omega^2x^2$

Mean $x = 0 \rightarrow P.E = 0$

$x = \pm A \rightarrow P.E = \frac{1}{2}m\omega^2A^2$

Total energy Mean $\rightarrow E = K.E + P.E = \frac{1}{2}m\omega^2A^2 + 0$

$E = \frac{1}{2}m\omega^2A^2$

extreme $\rightarrow E = K.E + P.E = 0 + \frac{1}{2}m\omega^2A^2$

$E = \frac{1}{2}m\omega^2A^2$

$E =$ constant $\Rightarrow E \propto \omega^2$, $E \propto \frac{1}{T^2}$

$\rightarrow$ Model ①

1) eq. of motion of particle executing SHM is $\frac{d^2y}{dt^2} + \alpha y = 0$, $T = ?$

$a = -\omega^2 y \rightarrow \frac{dv}{dt} = -\omega^2 y$

$\Rightarrow \frac{d}{dt}\left(\frac{dy}{dt}\right) = -\omega^2 y \Rightarrow \left.\begin{array}{l} \frac{d^2y}{dt^2} + \omega^2 y = 0 \\ \Rightarrow \frac{d^2y}{dt^2} + \alpha y = 0 \end{array}\right\}$

$\omega^2 = \alpha$

$\omega = \sqrt{\alpha}$

$\frac{2\pi}{T} = \sqrt{\alpha} \Rightarrow T = \frac{2\pi}{\sqrt{\alpha}}$.

$n = \sqrt{\alpha}/2\pi$

2) eq. of motion $a + 16\pi^2 x = 0 \Rightarrow T = ?$

$\left.\begin{array}{l} a + \omega^2 x = 0 \\ a + 16\pi^2 x = 0 \end{array}\right\} \Rightarrow \omega^2 = 16\pi^2 \Rightarrow \omega = 4\pi \Rightarrow \frac{2\pi}{T} = 4\pi \Rightarrow T = \frac{1}{2}$

$T = 0.5$ sec

$n = 2$ HZ

3) what is the velocity of the particle of SHM at a displacement $\pm A/2$.

$v = \omega\sqrt{A^2 - y^2}$

$y = \pm A/2 \rightarrow v = \omega\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}}{2}A\omega = \frac{\sqrt{3}}{2}$ max velocity.

$v = 0.866$ max velocity

$v = 86.6\%$ max velocity.

4) $y = 0.1\sin 100\pi(t + 0.05)$ what is T, n, A, ϕ, initial displacement.

$y = A\sin(\omega t + \phi)$

Amplitude $A = 0.1$

phase $\phi = 100\pi \times 0.05 = 5\pi = 900°$

$\omega = 100\pi \Rightarrow \frac{2\pi}{T} = 100\pi \Rightarrow T = \frac{1}{50}$ sec

$n = 50$ HZ.

$t = 0 \Rightarrow y = 0.1\sin(0 + 5\pi) = 0.1 \times 0 = 0$.

$y = a(\sin\omega t + \cos\omega t)$, what is amplitude.

$= \frac{a\sqrt{2}}{\cancel{\sqrt{2}}}\left[\sin\omega t \cdot \frac{1}{\sqrt{2}} + \cos\omega t \cdot \frac{1}{\sqrt{2}}\right] = \frac{a\sqrt{2}}{\cancel{\sqrt{2}}}\left[\sin\omega t \cos 45^\circ + \cos\omega t \cdot \sin 45^\circ\right]$

$y = \frac{a\sqrt{2}}{\cancel{\sqrt{2}}}\sin(\omega t + 45^\circ) = A\sin(\omega t + \phi)$

$\therefore A \rightarrow a\sqrt{2} \rightarrow 1.414\,a$

$\rightarrow 141.4\%\,a$

$y = 0.02\sin\left[\frac{2\pi}{0.01}t - \frac{2\pi}{0.30}x\right]$ find velocity of the wave,

$y = A\sin(\omega t + \phi)$

velocity $V = A\omega$ (or) $V = n\lambda$

$\omega = \frac{2\pi}{0.01}$ $\quad \frac{2\pi}{0.30} = k$ $\quad V = 100 \times 0.30$

$T = 0.01$ $\quad \frac{2\pi}{0.30} = \frac{2\pi}{\lambda}$ $\quad V = 30$ m/sec

$n = 100$ $\quad \lambda = 0.30$

→ A particle is executing linear SHM of amplitude A. when the displacement is half the amplitude the fraction of K.E is. $\frac{1}{5}$ $\frac{3}{4}$ $\frac{1}{2}$ $\frac{1}{4}$

$x = A/2 \longrightarrow K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\omega\sqrt{A^2 - x^2}\right)^2$

$K.E = \frac{1}{2}m\omega^2(A^2 - x^2)$

$x = A/2 \longrightarrow K.E = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

$= \frac{1}{2}m\omega^2 \frac{3A^2}{4}$

$= \frac{3}{4} \times \frac{1}{2}m\omega^2A^2$

$K.E\ \frac{3}{4}$ (Max energy) $\Rightarrow 75\%$

Model ② :-

→ A pendulum which is suspended in a lift moving upwards with an acceleration to g'. then it's period becomes ——

$T_1 = 2\pi\sqrt{\frac{l}{g}}$ ⇒ not moving

up ⇒ $a=g$ ⇒ $T = 2\pi\sqrt{\frac{l}{g+a}}$ ⇒ $T = 2\pi\sqrt{\frac{l}{g+g}}$

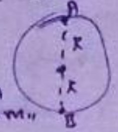
$T = 2\pi\sqrt{\frac{l}{2g}}$

$T = \frac{1}{\sqrt{2}} \cdot 2\pi\sqrt{\frac{l}{g}} \Rightarrow T = \frac{T_1}{\sqrt{2}}$ ↓ $\sqrt{2}$ times.

n is ↑ $\sqrt{2}$ times.

→ If a hole is drilled along the diameter of the earth and a stone is dropped into the hole, then T = ——

$T = 2\pi\sqrt{\frac{l}{g}}$.



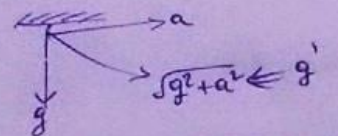
~~at the center g = 0 ⇒ T = ∞, n = 0~~

$l = R = 6400\ km$
$= 6400000\ m$,

$T = 2\pi\sqrt{\frac{6400000}{9.8}} = 5078\ sec = \frac{5078}{60} = 84.6\ min$ → to complete one cycle (A–B–A)

$T = 42.3\ min$ → to go other side (B). (half cycle).

→ A pendulum suspended from the roof of a trolley which moves in a horizontal ~~surface~~ direction with an acceleration 'a' then the time period $T = 2\pi\sqrt{\frac{l}{g'}}$, wher $g' =$ ——



→ pendulum oscillate inbetween $x = -A$ and $x = +A$. Time taken for it to go from 0 to $A/2$ is T_1 and $A/2$ to A is T_2 then.

(a) $T_1 = 2T_2$ (b) $T_1 = T_2$ (c) $T_1 < T_2$ (d) $T_1 > T_2$.

→ The length of a simple pendulum is increased by 44% then % increase in T is ?

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$

$l_2 = l_1 + 44\%\ l_1$

$l_2 = \frac{144}{100} l_1$

$\frac{T_1}{T_2} = \sqrt{\frac{100}{144}} \rightarrow T_2 = \sqrt{\frac{144}{100}}\, T_1$

$T_2 = \frac{12}{10} T_1 = 1.2\, T_1 = T_1 + 20\%\, T_1$

20% increase

i.e 0.2 times.

→ pendulum of length l_1 has Time period T_1 and l_2 it is T_2. What is the Time period of pendulum of leng $l_1 + l_2$.

$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$ $T_2 = 2\pi\sqrt{\frac{l_2}{g}}$, $T = 2\pi\sqrt{\frac{l_1+l_2}{g}}$

$T^2 = \frac{4\pi^2 l_1}{g^2} + \frac{4\pi^2 l_2}{g^2}$

$T^2 = T_1^2 + T_2^2 \Rightarrow T = \sqrt{T_1^2 + T_2^2}$

A pendulum having Time period T. If it's length increased by 10 cm then it's period is T_1. If it's length decreased by 10 cm then it's period is T_2. Then relation between T, T_1, T_2

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad T^2 = 4\pi^2\frac{l}{g}$$

$$T_1 = 2\pi\sqrt{\frac{l+10}{g}} \qquad T_1^2 = 4\pi^2\left(\frac{l+10}{g}\right)$$

$$T_2 = 2\pi\sqrt{\frac{l-10}{g}} \qquad T_2^2 = 4\pi^2\left(\frac{l-10}{g}\right)$$

$$\Rightarrow T_1^2 + T_2^2 = \frac{4\pi^2(l+10)}{10} + \frac{4\pi^2(l-10)}{g}$$

$$\underline{T_1^2 + T_2^2 = 2T^2}$$

→ if a ~~spring~~ length of the pendulum ↑ n times then Time period ⇒ $\sqrt{n}$ ↑

↓ " ⇒ $\sqrt{n}$ ↓

if a spring is cut into n parts, then time period ⇒

$$T = 2\pi\sqrt{\frac{m}{k}}, \quad K = \frac{F}{x}$$

$$\text{if } x \Rightarrow \frac{x}{n} \Rightarrow K = \frac{nF}{x} \rightarrow nk \Rightarrow T = 2\pi\sqrt{\frac{m}{nk}}$$

$$T \Rightarrow \frac{2\pi\sqrt{m/x}}{\sqrt{n}} \Rightarrow \underline{\frac{T}{\sqrt{n}}}$$

Heat & Thermodynamics

Heat → It is the form of energy which can transfer from high temp body to low temp body.

feeling ~~xxxx~~ Hot → when Heat entering into the body

feeling cold → when heat loss (exit).

1cal = 4.2 Joule

Heat transfer

1) conduction process → transfer of heat by collisions of particles in a medium. Ex:- Solid

2) convection process → transfer of heat due to actual motion of particles. Ex:- Cooking Rice

3) Radiation → Heat through empty space.
Ex: body get warm infront of a fire.

Temperature

It is the measure of degree of hotness (or) coldness of a gas.

(or)

It is the physical quantity that tells whether the Body is in thermal equillibrim (or) not.

scale	low	highest
Celcius	$0^\circ C$	$100^\circ C$
Farenheit	$32^\circ F$	$212^\circ F$
Kelvin	273 K	373 K

$F = \frac{9}{5}C + 32$

$T = 273 + C.$

→ when a matter is heated, there is a change (↑or↓) in volume and pressure.

Pressure → force applied by a gas on unit Area.

$\boxed{P = \frac{F}{A}}$ → N/m^2 → pascal

1 PSi = 6894.7 pascal.

$\boxed{P = hdg}$ ⇒ Pαh

$d = 13.6 \times 10^3 \ kg/m^3$ Mercury density

Mercury ⇒ can't react with any. freeze $-38.8^\circ C$, boils $356^\circ C$,

Atmospheric pressure

pressure applied by Atmospheric air on the surface of the earth.

At see level $h = 76 cm = 760 mm = 0.76 m$ of Hg
⇒ $P = 101292.8$ pascal $= 1 atm$

At Mount everest $h = 25.3 cm = 253 mm = 0.253 m$ of Hg
⇒ $P = 33719$ pascal $= \frac{1}{3} atm$.

$P_{sea} > P_{everest}$, Pacific Ocean → 32 atm.

STP condition

T = 273 kelvin

P = 1atm

V = 22.4 litre
= $0.0224 \ m^3$

NTP condition

$T = 27^\circ C$ → 300K

P = 1 atm.

V = ? changes.

Gas laws $\Longrightarrow$ Gas behaviour $P_r\, V_c$

Boyle's law

$T =$ constant

$P \propto \frac{1}{V}$ inversely

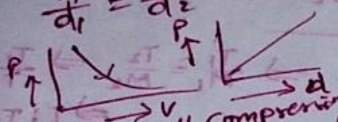
$PV =$ constant

$\boxed{P_1 V_1 = P_2 V_2}$

and

$P \propto d$ (density)

$\frac{P_1}{d_1} = \frac{P_2}{d_2}$



Rubber ball compression, piston

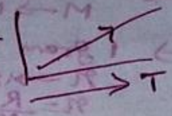
Charles 1st

$P =$ constant

$V \propto T$

$\frac{V}{T} =$ constant

$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$



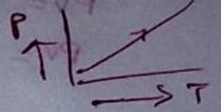
Charles 2nd

$V =$ constant

$P \propto T$

$\frac{P}{T} =$ const

$\boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}$



$\rightarrow$ pressure coefficient of gas $\quad \gamma = \frac{P_2 - P_1}{P_1 t_2 - P_2 t_1}$

" " volume $\quad \gamma = \frac{V_2 - V_1}{V_1 t_2 - V_2 t_1}$, combined temp $= \frac{M_1 T_2 + M_2 T_1}{M_1 + M_2}$

combined pressure $P = \frac{P_1 T_2 + P_2 T_1}{P_1 + P_2}$

$\rightarrow$ At $30^\circ C$ the volume of the gas is $200 cm^3$. At constant pressure the gas is heated to $100^\circ C$. find final volume?

$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{200}{303} = \frac{V_2}{373} \Rightarrow V_2 = 246\ cm^3$

$\rightarrow$ A gas at $10^\circ C$, if its volume is doubled then final temp?

$\frac{V_1}{283} = \frac{2V_1}{T_2} \rightarrow T_2 = 2 \times 283 = 566\ K = 293^\circ C$

$\rightarrow$ At constant pressure, the ratio of increase in volume of an ideal gas per degree rise in temp to it's original volume is $\frac{\left(\frac{dV}{dT}\right)}{V_1} = ?$

$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1} \Rightarrow \frac{V_2}{V_1} - 1 = \frac{T_2}{T_1} - 1$

$\Rightarrow \frac{V_2 - V_1}{V_1} = \frac{T_2 - T_1}{T_1}$

$\Rightarrow \frac{dV}{V_1} = \frac{dT}{T_1} \Rightarrow \frac{dV}{dT\, V_1} = \frac{1}{T_1}$

$\Rightarrow \frac{dV/dT}{V_1} = \frac{1}{T_1}$

$\rightarrow$ A gas is heated at constant volume from $27^\circ C$ to $87^\circ C$, what is the % increase in pressure. $\left(\frac{dP}{P_1}\right) = ?$

$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1}$

$\Rightarrow \frac{P_2}{P_1} - 1 = \frac{T_2}{T_1} - 1 \Rightarrow \frac{P_2 - P_1}{P_1} = \frac{T_2 - T_1}{T_1}$

$\Rightarrow \frac{dP}{P_1} = \frac{60K}{300K} \Rightarrow \frac{dP}{P_1} = \frac{1}{5} = 0.2 = 20\%$

(or) $dP = P_2 - P_1 = \frac{P_1}{5}$, $P_2 = P_1 + \frac{P_1}{5}$, 20%

$\rightarrow$ If the pressure of a gas is increased by 20% at constant temp, what is the % change in it's volume.

$P_2 = P_1 + 20\% P_1 \quad P \propto \frac{1}{V} \Rightarrow P_1 V_1 = P_2 V_2$

$V_2 = ?$

$\Rightarrow P_1 V_1 = (P_1 + 20\% P_1) V_2$

$\Rightarrow P_1 V_1 = \frac{120}{100} P_1 V_2$

$\Rightarrow V_2 = \frac{100}{120} V_1 = \frac{5}{6} V_1 = \left(1 - \frac{1}{6}\right) V_1$

$V_2 = V_1 - \frac{V_1}{6} = V_1 - 16.7\%\, V_1$

$\rightarrow$ The pressure of a gas is 80 cm of Mercury. Now 25% of the gas is also introduced in the same vessel at the same temp. What will be the final pressure of the gas in the vessel.

$V =$ Constant, $T =$ Constant

$P \propto d \Rightarrow P \propto M \Rightarrow \frac{P}{M} =$ constant $\Rightarrow \frac{P_1}{M_1} = \frac{P_2}{M_2}$

$P_1 = 80 cm$

$M_2 = M_1 + 25\% M_1$

$\frac{80}{M_1} = \frac{P_2}{\frac{125}{100} M_1} \Rightarrow P_2 = 80 \times \frac{125}{100} = 100 cm$

$P_2 = 80 + 20$

20% ext

Ideal gas
↳ Imaginar gas which obey's all gas laws at all temp & pressure.

$\frac{PV}{T}$ = constant (R) ↳ R = 8.31 Joule/kelvin/mole universal gas constant.

$PV = RT$ → for 1 mole gas

$PV = nRT$ → " n " "

$PV = \frac{m}{M}RT$ → m → mass of gas
M → Molecular weight of gas.

$PV = rT$ → 1 gram mass of gas
r → general gas constant.
$r = \frac{R}{M}$.

→ gases O_2 and H_2 have ~~the~~ same mass, volume, pressure then temp ratios is ———

$PV = \frac{m}{M}RT \Rightarrow \frac{T}{M} = \text{const} \Rightarrow \frac{T_1}{M_1} = \frac{T_2}{M_2} \Rightarrow \frac{T_1}{T_2} = \frac{M_1}{M_2} = \frac{32}{2}$

$\Rightarrow T_1 : T_2 = 16:1$

→ A vessel having 8 gm of mass at 400K with a hole on top. After some time, the pressure is halved and temp is changed to 300k., then the mass of air that is escaped is.

$PV = \frac{m}{M}RT$

V, M, R → const

$P = mT$

$\frac{P_1}{P_2} = \frac{m_1 T_1}{m_2 T_2} \Rightarrow \frac{P_1}{P_1/2} = \frac{8 \times 400}{m_2 \times 300}$

$\Rightarrow 2 = \frac{32}{3 m_2} \Rightarrow m_2 = \frac{16}{3} = 5.33$ gm

mass escaped $= 8 - 5.33$
$= 2.67$ gm escap

→ 16 gm of O_2, x gm of H_2 occupied same volume at same temp, pressure. Find x.

$\frac{m}{M} = \text{const} \Rightarrow \frac{m_1}{M_1} = \frac{m_2}{M_2} \Rightarrow \frac{16}{32} = \frac{x}{2} \Rightarrow x = 1$ gm.

→ 1 litre of gas at 10°c is heated such that it's V, P are doubled than final temp?

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{P_1 \times 1}{283} = \frac{2P_1 \times 2}{T_2}$

$\Rightarrow T_2 = 4 \times 283 = 1132$ K $= 859$°c.

Boltzman constant $K = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} \Rightarrow K = 1.38 \times 10^{-23}$ J/k

constant for one molecule of gas.

→ ideal gas occupies V, P, T. If the mass of each molecule is m. What is the equation for density in Boltzman constant,

$PV = nRT \Rightarrow P \cdot \frac{m}{d} = \frac{m}{M}RT$

$K = \frac{R}{N} \Rightarrow R = KN$

$\Rightarrow \frac{P}{d} = \frac{KNT}{M}$

$m = \frac{M}{N} = \frac{\text{Molecular weight}}{\text{Avagadro number}}$

m → mass of one molecule,

$\Rightarrow d = \frac{MP}{KNT}$

$d = \frac{mP}{KT}$.

Thermodynamics $\Rightarrow$ Mechanical work $\Leftrightarrow$ Heat
(P.E, K.E)

$W \propto H \Rightarrow W = JH$, $J = 4.2$ Joule/cal
Mechanical equivalent

1) if body falls from 'h' height it's P.E is converted into heat
$W = P.E = mgh$, Heat $H \propto mt$
$H = Smt$, $S \to$ specific heat, $m \to$ mass, $t \to$ temperature.

$W = JH$
$mgh = JSmt \Rightarrow t = \frac{gh}{JS} \Rightarrow t \propto h$

2) A bullet fired into the wood then K.E into heat.
$K.E = H$
$\frac{1}{2}mv^2 = JSmt \Rightarrow t = \frac{v^2}{2JS} \Rightarrow t \propto v^2$

3) 'm' mass of ice (solid) falls from height 'h', if H heat is produced to melts the ice (liquid).
$mgh = JmL \Rightarrow h = \frac{JL}{g}$

$\to$ Zeroth Law $\to$ thermal equilibrium $\to$ prove of temperature existance.

$\to$ 1st Law $\to$ Conservation of energy
$dQ = dU + dW$ — heat, internal energy, work.
$T\,dS = C_v\,dT + P\,dv$
$dS \to$ entropy change (disorder); $C_v \to$ specific Heat at constant volume.

$\to$ Isothermal process (slow process) (conducting wall): $T =$ constant, $dT = 0$, $dU = 0$ $\Rightarrow P \propto \frac{1}{V} \Rightarrow PV =$ constant (Boyle's)
$\to dQ = dW = P\,dv$, heat = work
Ex: conversion of water at 100°C into steam at 100°C, ice at 0°C into water 0°C

$\to$ Adiabatic process (sudden process) (insulating wall): $Q =$ constant, $dQ = 0$ $\to dU = -dW$, $dW = -dU$ Ex: Tube burst
Boyle's law not holds here and $PV^{\gamma} =$ const
$TV^{\gamma-1} =$ const
$P^{1-\gamma}T^{\gamma} =$ const
$\gamma = \frac{C_p}{C_v}$ Specific heat Ratio.

$\gamma = \frac{5}{3} = 1.6$ for ideal gas, $\gamma = 1.67$ Monoatomic gas.
$= 7/5 = 1.4$ for diatomic gas
$= 1.33$ for Triatomic gas

$\to$ Isobaric process: $P =$ constant
$\to$ Isochoric process: $V =$ constant $\to dV = 0$, $dW = 0 \Rightarrow dQ = dU$, heat = internal energy.
$\to$ cyclic process: initial = final — initial ice — water — vapour
$U =$ constant $\Rightarrow dU = 0 \Rightarrow dQ = dW$
$\to$ Reversible process: backward process also.
$\to$ irreversible process: only forward process. Ex: buring wood.

$$\gamma_{mixed} = \frac{\frac{n_1\gamma_1}{\gamma_1 - 1} + \frac{n_2\gamma_2}{\gamma_2 - 1}}{\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}}$$

Specific Heat → constant for 1 gram.

It is the amount of heat required to rise the temp of 1 gram of gas through $1^\circ C$ or $1K$

$dQ \propto m\,dT$

$dQ = c\,m\,dT$

$c = \dfrac{dQ/dT}{m}$ → Joule/kelvin.mole.

Molar specific heat → constant for 1 mole

It is the amount of heat required to rise the temp of 1 mole of gas through $1^\circ C$.

$C = \dfrac{dQ/dT}{n}$

spec $c = 1$ cal/gm → water

$L = 80$ cal/gm → latent heat of fusion of water

$= 540$ cal/gm → latent heat of vaporization of water at $100^\circ C$.

→ 2000 J of heat is given to a gas, when it's volume increases by $3\times10^{-3}\,m^3$ at a constant pressure of 10^5 pa. The increase in internal energy?

$dU = dQ - P\,dV$
$= 2000 - (10^5 \times 3\times10^{-3})$
$= 2000 - 300 = 1700$ Joule.

→ A gas obeys $PV^{3/2} = $ const in Adiabatic process. If it's volume is halfed, what is final temp.

$PV^{3/2} = $ const $\Rightarrow TV^{\gamma-1} = $ const

$TV^{3/2-1} \Rightarrow TV^{1/2} = $ const

$\Rightarrow T_1\sqrt{V_1} = T_2\sqrt{V_2}$

$\Rightarrow \dfrac{T_1}{T_2} = \sqrt{\dfrac{V_2}{V_1}} = \sqrt{\dfrac{V_1/2}{V_1}} = \dfrac{1}{\sqrt2}$

$\Rightarrow T_2 = \sqrt2\,T_1 = 1.414\,T_1$

$T_2 = T_1 + \underline{41.4\%\,T_1}$, increases

→ A gas is suddenly compressed to $\frac{1}{3}$ of its volume. If the initial temp is $34^\circ C$ and $\gamma = 1.5$. find final temp.

$V_2 = \frac{V_1}{3}$

$T_1 = 307\,K$

$\gamma = 1.5$

$T_2 = ?$

$TV^{\gamma-1} = $ const

$\dfrac{T_1}{T_2} = \left(\dfrac{V_2}{V_1}\right)^{\gamma-1} \Rightarrow \dfrac{307}{T_2} = \left(\dfrac{V_1/3}{V_1}\right)^{1.5-1} = \left(\dfrac{1}{3}\right)^{0.5} = \dfrac{1}{\sqrt3}$

$\Rightarrow T_2 = \sqrt3 \times 307 = 531.7\,K$

$T_2 = 258^\circ C$.

→ for a gas $P \propto T^3$ find $\gamma = ?$ $P \propto T^3 \Rightarrow P \propto \left(\frac{PV}{R}\right)^3 \Rightarrow P^2V^3 = $ const $\Rightarrow PV^{3/2} = $ const, $\gamma = 3/2$.

→ ~~1 mole~~ 1 mole of ideal gas ($\gamma = 5/3$) and 1 mole of diatomic gas ($\gamma = 7/5$) are mixed together then resultant $\gamma = ?$

$$\gamma = \frac{\dfrac{1\times 5/3}{5/3-1} + \dfrac{1\times 7/5}{7/5-1}}{\dfrac{1}{5/3-1} + \dfrac{1}{7/5-1}} = \frac{\frac{5}{2}+\frac{7}{2}}{\frac{3}{2}+\frac{5}{2}} = \frac{12}{8} = \frac{3}{2}$$

→ Heat engine → Device which convert heat into work.

efficiency $\eta = \dfrac{W}{Q_1} = 1 - \dfrac{Q_2}{Q_1}$ or $1 - \dfrac{T_2}{T_1}$

always $\eta < 1 \Rightarrow$ no perfect engine
no engine can convert 100% heat into 100% work.

→ cool engine → engine which transfer low temp to high temp body.
(refrigerator)

performance $K = \dfrac{Q_2}{W} = \dfrac{Q_2}{Q_1 - Q_2}$ (or) $\dfrac{T_2}{T_1 - T_2}$

$K = \dfrac{1-\eta}{\eta}$ (or) $\eta = \dfrac{1}{1+K}$

Large K value ⇒ means better engine (refrigerator)

Modernphysics

→ photo electric effect ⟶ emission of $\bar{e}s$ from met

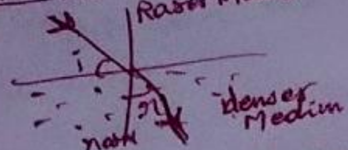
$$h\nu = h\nu_0 + \underbrace{\tfrac{1}{2}mv^2}_{K.E\ \text{of}\ \bar{e}s}$$

photons

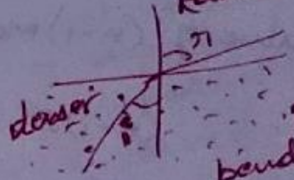
$$\tfrac{1}{2}mv^2 = h\nu - h\nu_0 = hc\left(\tfrac{1}{\lambda} - \tfrac{1}{\lambda_0}\right)$$

- **Laws** 1) photo current $\propto$ intensity of light
  2) velocity or energy of photo $\bar{e}s$ $\propto$ frequency of light.
  3) it is instantanous process ; $\bar{e}s$ ejected in 3 nanosec.

Refraction light



$i > r$
bending towards normal.



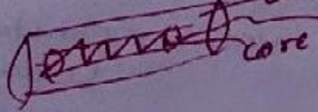
$i < r$
bending away from normal.

Refractive index $\mu = \frac{c}{v} = \frac{3\times10^8}{(\ \ )}$ $\mu \rightarrow$ no UNITS

$$\mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{1}{\sin i_c}$$ — $i_c \rightarrow$ critical angle.

→ optical fibers works on principle of total internal Reflection.



numerical aperture $= \sqrt{n_1^2 - n_2^2}$

The end ⟶